Homework 8

Due Thursday, November 12th at 11:59pm

Reread Chapter 1 of Schroeder's book *Thermal Physics*.

Problem 1 In the course of pumping up a bicycle tire, a liter of air at atmospheric pressure is compressed adiabatically to a pressure of 7 atm. (Air is mostly diatomic nitrogen and oxygen.)

(a) Derive Schroeder's Eq. (1.40) from his Eq. (1.39).

(b) What is the final volume of this air after compression?

(c) How much work is done in compressing the air?

(d) If the temperature of the air is initially 300 K, what is the temperature after compression?

Problem 2 In a Diesel engine, atmospheric air is quickly compressed to about 1/20 of its original volume. Estimate the temperature of the air after compression, and explain why a Diesel engine does not require spark plugs.

Problem 3 (The Atmosphere Again)
In Problem 2 of last week's homework you calculated the pressure of earth's atmosphere as a function of altitude, assuming constant temperature. Ordinarily, however, the temperature of the bottommost 10-15 km of the atmosphere (called the tropo- sphere) decreases with increasing altitude, due to heating from the ground (which is warmed by sunlight). If the temperature gradient $|dT/dz|$ exceeds a certain critical value, convection will occur: Warm, low-density air will rise, while cool, high-density air sinks. The decrease of pressure with altitude causes a rising air mass to expand adiabatically and thus to cool. The condition for convection to occur is that the rising air mass must remain warmer than the surrounding air despite this adiabatic cooling.

(a) Show that when an ideal gas expands adiabatically, the temperature and pressure are related by the differential equation

$$\frac{dT}{dP} = \frac{2}{d+2}\frac{T}{P}$$

where d is the number of degrees of freedom of the gas.

(b) Assume that dT/dz is just at the critical value for convection to begin, so that the vertical forces on a convecting air mass are always approximately in balance. Use the result of Problem 2 (b) from last week to find a formula for dT/dz in this case. The result should be a constant, independent of temperature and pressure, which evaluates to approximately -10° C/km. This fundamental meteorological quantity is known as the dry adiabatic lapse rate.

Problem 4 (Measuring Heat Capacities)
To measure the heat capacity of an object, all you usually have to do is put it in thermal contact with another object whose heat capacity you know. As an example, suppose that a chunk of metal is immersed in boiling water (100°C), then is quickly transferred into a Styrofoam cup containing 250 g of water at 20°C. After a minute or so, the temperature of the contents of the cup is 24°C. Assume that during this time no significant energy is transferred between the contents of the cup and the surroundings. The heat capacity of the cup itself is negligible.

(a) How much heat is lost by the water?

(b) How much heat is gained by the metal?

(c) What is the heat capacity of this chunk of metal?

(d) If the mass of the chunk of metal is 100 g, what is its specific heat capacity?

Problem 5 (Gaseous Engines)
An ideal gas (1 mol) is the working substance in an engine that operates on the cycle shown in the Figure. Processes BC and DA are reversible and adiabatic.

(a) Is the gas monatomic, diatomic, or polyatomic?

(b) What is the engine efficiency?