Homework 9

Due Monday, November 23rd at 2pm

Begin reading Chapter 2 of Schroeder's book *Thermal Physics*. For the remainder of the course we are going to be using the ideas behind probabilities more and more. To help get you oriented, I've added a short (7 page) reading on how to calculate averages and standard deviations using probabilities to our course site here.

Problem 1 (Cooling a thermos)
An insulated Thermos contains 130 g of water at 80°C. You put in a 12 g ice cube at 0°C to form a system of *ice + original water*. (a) What is the equilibrium temperature of the system? What are the entropy changes of the water that was originally the ice cube (b) as it melts and (c) as it warms to the equilibrium temperature? (d) What is the entropy change of the original water as it cools to the equilibrium temperature? (e) What is the net entropy change of the *ice + original water* system as it reaches the equilibrium temperature?

Problem 2 (Age distribution)
For the distribution of ages in Section 1.3.1 of the new handout:
(a) Compute $\langle j^2 \rangle$ and $\langle j \rangle^2$.
(b) Determine Δj for each j, and use Eq. [1.11] to compute the standard deviation.
(c) Use your results in (a) and (b) to check Equation [1.12].

Problem 3 (Rock and cliff)
(a) Find the standard deviation of the distribution of Example 1.1 of the handout.
(b) What is the probability that a photograph, selected at random, would show a distance x more than one standard deviation away from the average?

The next two problems are very mathematical, and you may have already studied them in Math Methods, but they are so important that it is fine if you do them a couple of times. If they are old to you, use this opportunity to completely memorize the technique, which will be valuable to you so many times in the future.

Problem 4 (Gaussian Integral)
In class on Monday we will show that

$$\int_{-\infty}^{\infty} e^{-x^2} dx = \sqrt{\pi}.$$

(a) Use substitution to prove that

$$\int_{-\infty}^{\infty} e^{-ax^2} dx = \sqrt{\frac{\pi}{a}}.$$

This is the most useful integral you will encounter in your undergraduate career and probably for much longer. I highly recommend memorizing this result.
(b) Find the value of the integral

$$\int_{0}^{\infty} e^{-ax^2} dx.$$

[Hint: This value is best found without doing any further calculation.]
(c) Find the values of the integrals

$$\int_{-\infty}^{\infty} xe^{-ax^2}dx, \quad \int_{-\infty}^{\infty} x^3e^{-ax^2}dx, \quad \int_{-\infty}^{\infty} x^5e^{-ax^2}dx, \cdots .$$

[Hint: I'm asking you to do an infinite set of integrals, so again calculation would be ill advised.]
(d) Using your result from part (a), take the derivative of both sides of that equation with respect to a to find the value of the integral

$$\int_{-\infty}^{\infty} x^2e^{-ax^2}dx.$$

(e) Explain how you would find the integrals

$$\int_{-\infty}^{\infty} x^4e^{-ax^2}dx, \qquad \text{and} \qquad \int_{-\infty}^{\infty} x^6e^{-ax^2}dx.$$

Just words is enough for this part, you don't need to actually do the calculations.

Problem 5 (Gaussian practice)
Consider the **gaussian** distribution

$$\rho(x) = Ae^{-\lambda(x-a)^2},$$

where A, a, and λ are positive real constants.
(a) Use Eq. [1.16] of the handout to determine A. We call this normalizing the distribution.
(b) Find $\langle x \rangle$, $\langle x^2 \rangle$ and σ. What does σ characterize about this distribution?