Today

I. Last Time

II. Finish deriving consequences of the two postulates

III. Spacetime (Minkowski) Diagrams

I. Derived time dilation from Einstein's two postulates and a train thought-experiment:

"Moving clocks run slow" $\Delta t = \gamma \Delta t'$, where $\gamma = \dfrac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$.

Here we've taken the ground frame to be the S, that is, the unprimed frame. We've taken the train frame to be the S', that is, the primed frame.

We also began to derive the Length Contraction consiequence.

Which detector fires first?

(A) According to an observer on the train?

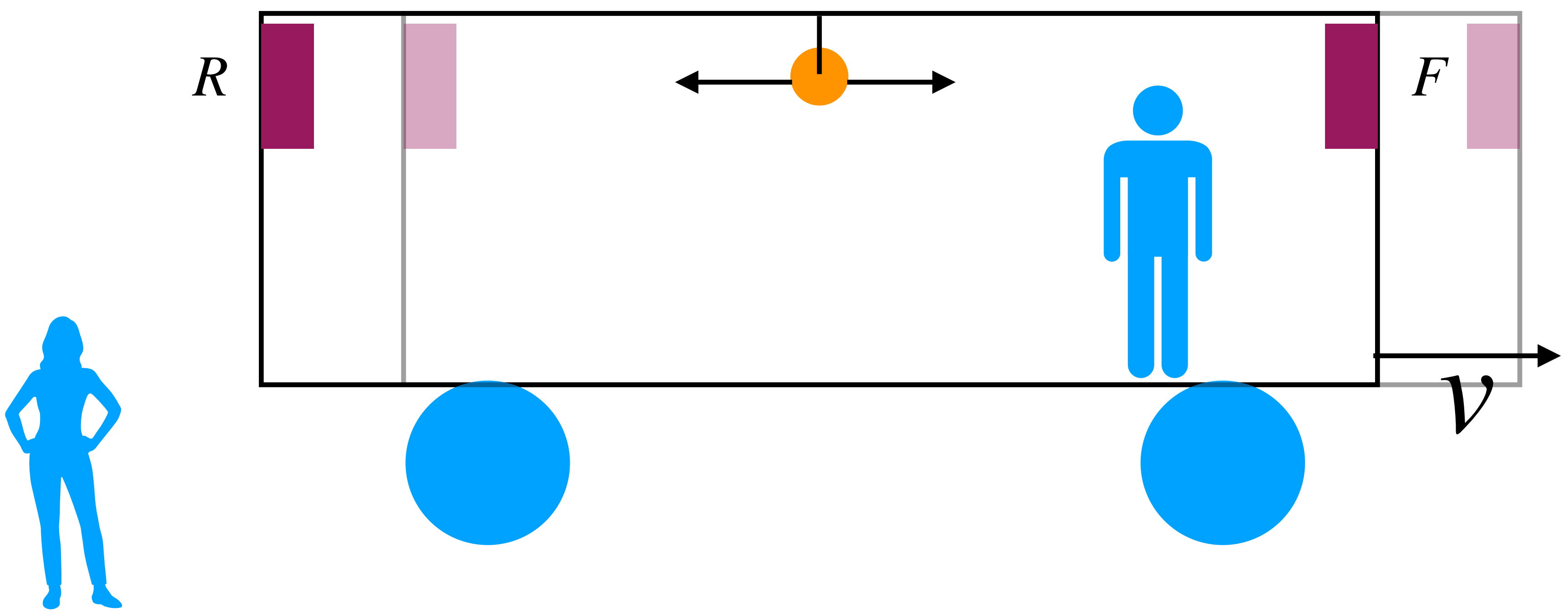
R and F fire simultaneously.

(B) According to an observer on the ground?

R fires first and F second.

The fact that they disagree has massive consequences. There is no universal notion of simultaneity!!!

Conclusion: (1) Two events simultaneous to one (inertial) observer, may not be to another.

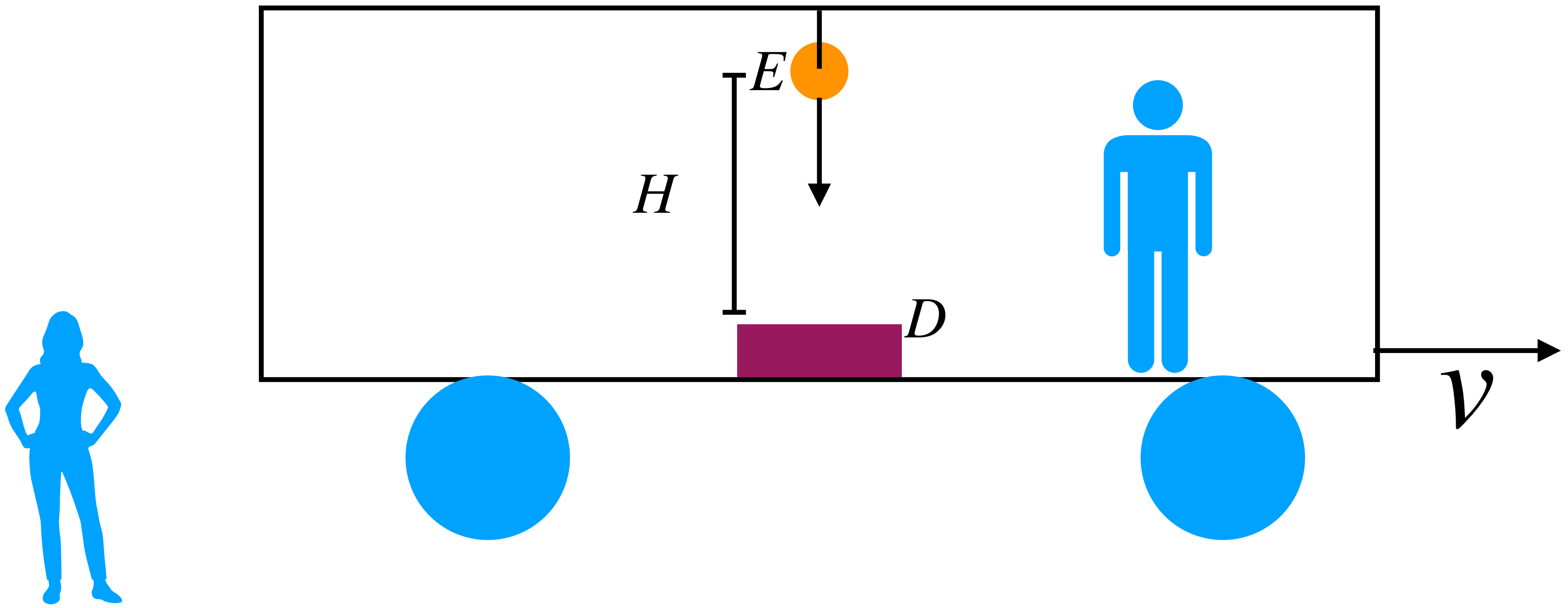


An observation: What you get after correcting for how long the message took to reach you. You could think of a f”custodian" attached to each reference frame.

II. (2) Time Dilation: How long between E and D?

(A) Observer on train: $\Delta t' = \dfrac{H}{c}$.

(B) Observer on the ground:



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(A) Observer on train: $\Delta t' = \dfrac{H}{c}$.

(B) Observer on the ground:

$$\Delta t = \frac{\sqrt{H^2 + v^2 \Delta t^2}}{c}.$$

Now, let's solve for Δt: $c^2 \Delta t^2 = H^2 + v^2 \Delta t^2$

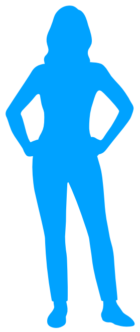
$$(c^2 - v^2)\Delta t^2 = H^2$$

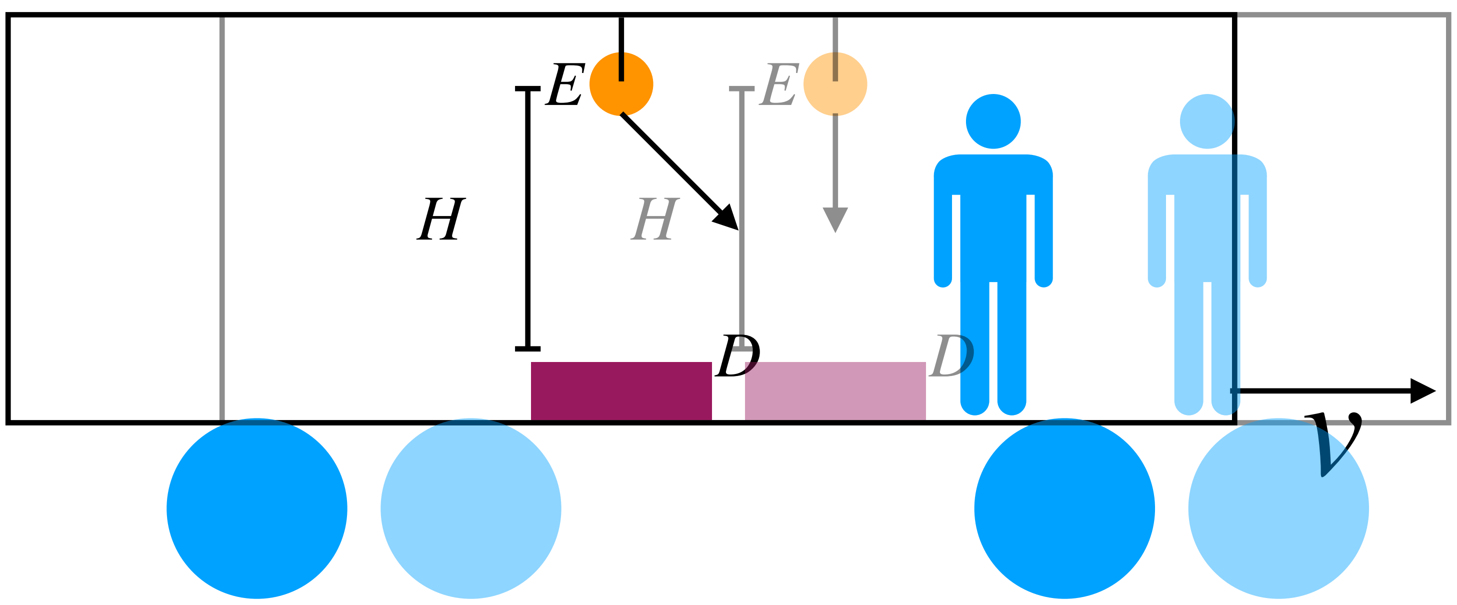
We always say that "Moving clocks run slow".

$$\left(1 - \frac{v^2}{c^2}\right)\Delta t^2 = \frac{H^2}{c^2} \implies \Delta t = \gamma \frac{H}{c} = \gamma \Delta t'$$

$$\gamma \equiv \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \geq 1$$

Slow means they have fewer clicks. In other words, you age more slowly when you are moving.





An example: $v = \frac{3}{4}c$, then $\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{1}{\sqrt{1 - \frac{9}{16}}} = \frac{1}{\sqrt{\frac{7}{16}}} = \frac{4}{\sqrt{7}}$

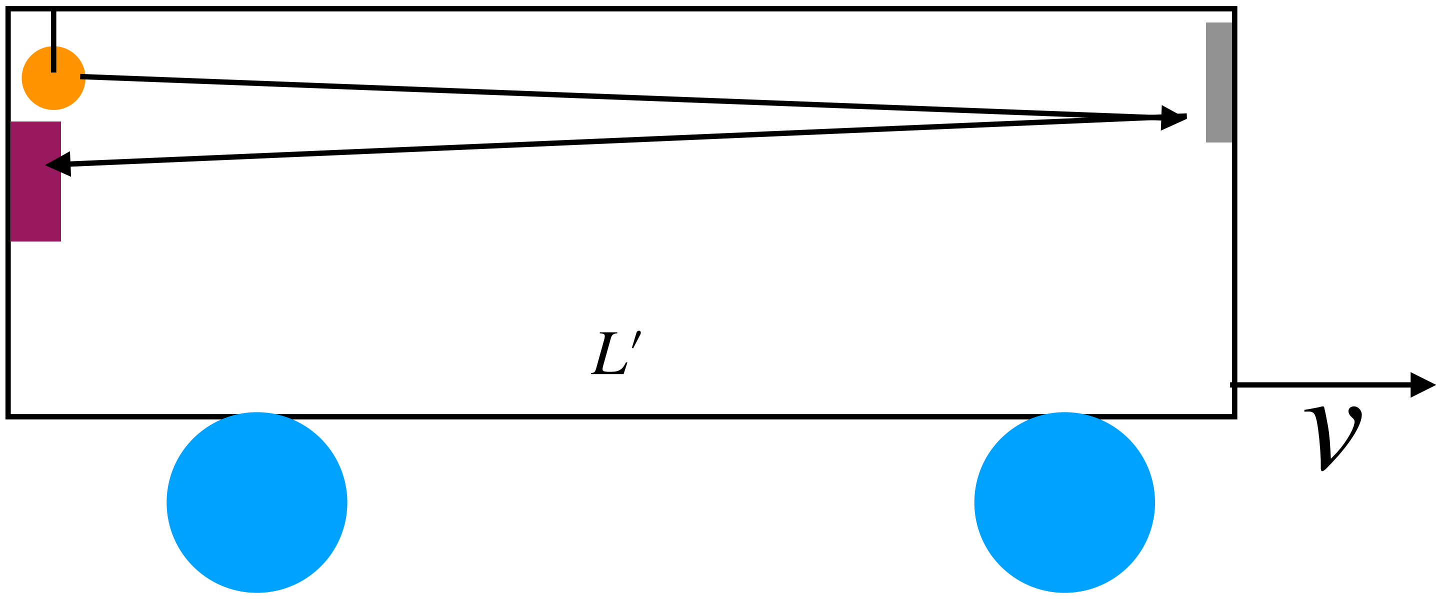
So, $\gamma \approx 1.5$, and after one second according to the train observers, a ground observer will measure 1.5 seconds.

(3) <u>Length Contraction (Lorentz Contraction):</u> Again we'll use a train experiment.

Question: How long does the round trip take?

(A) On train;

$$\Delta t' = \frac{2L'}{c}$$



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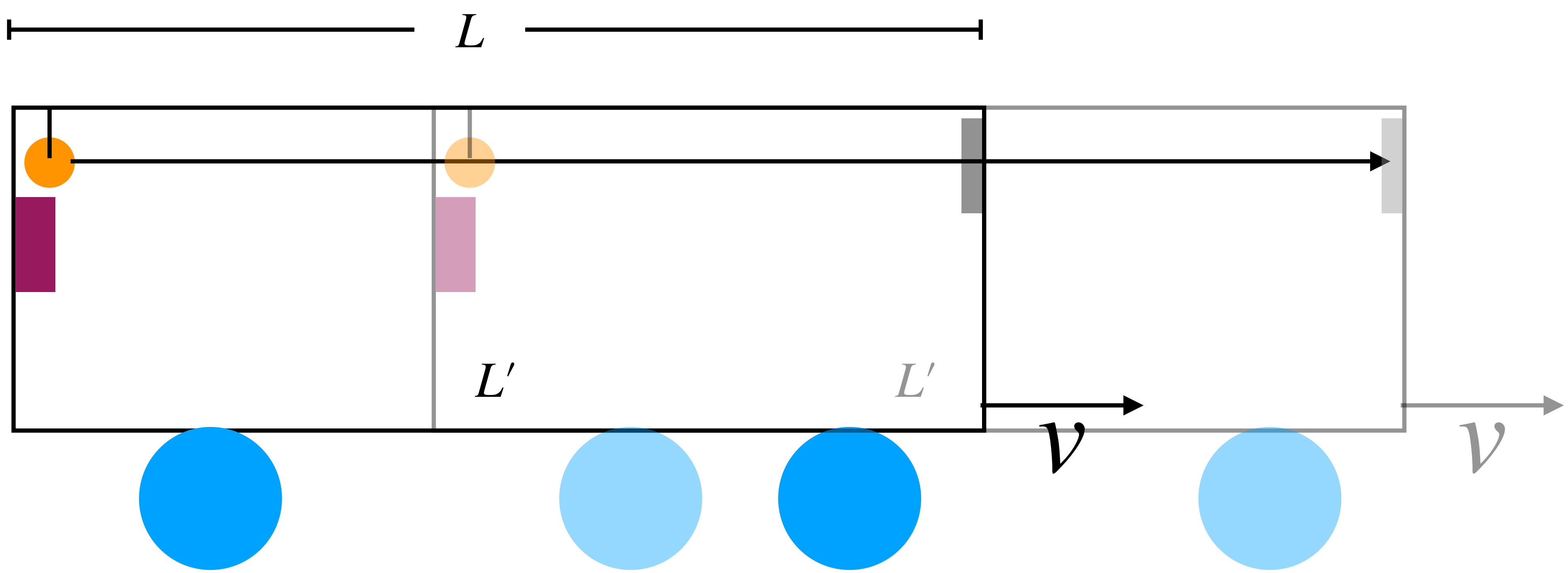

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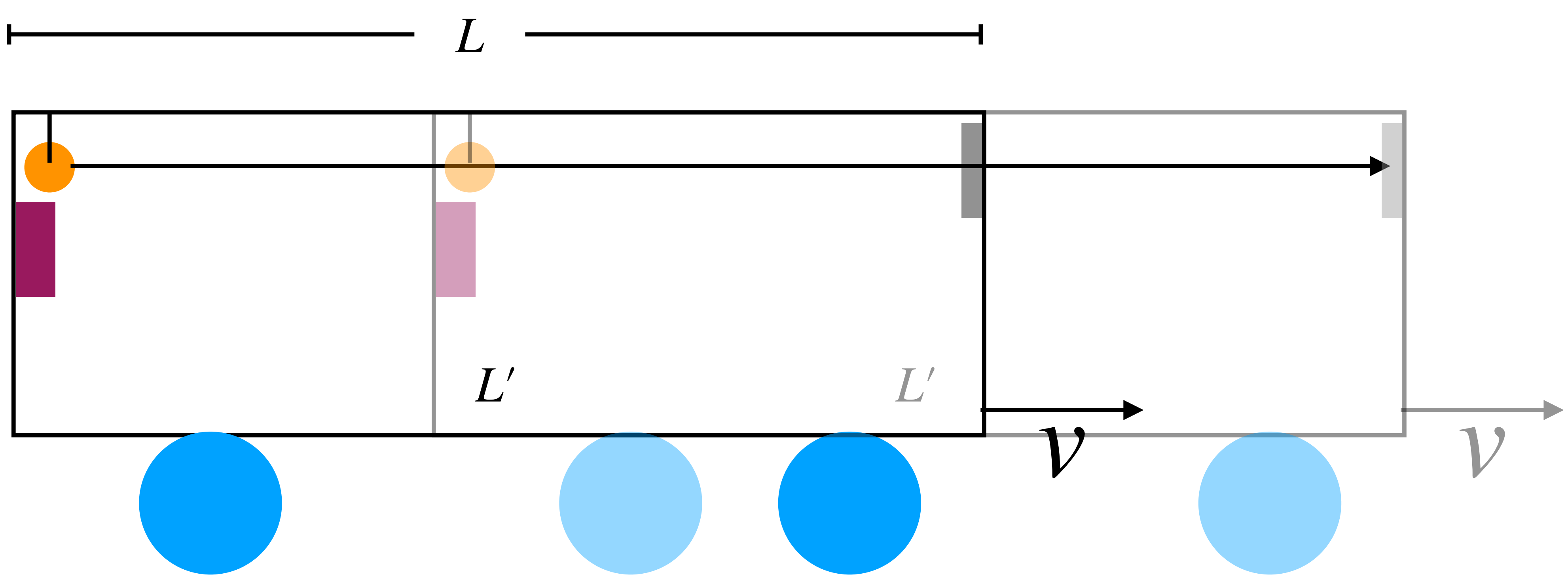
A little algebra: $\Delta t_1 \left(1 - \dfrac{v}{c}\right) = \dfrac{L}{c}$ or $\Delta t_1 = \dfrac{L}{c}\left(1 - \dfrac{v}{c}\right)^{-1}$



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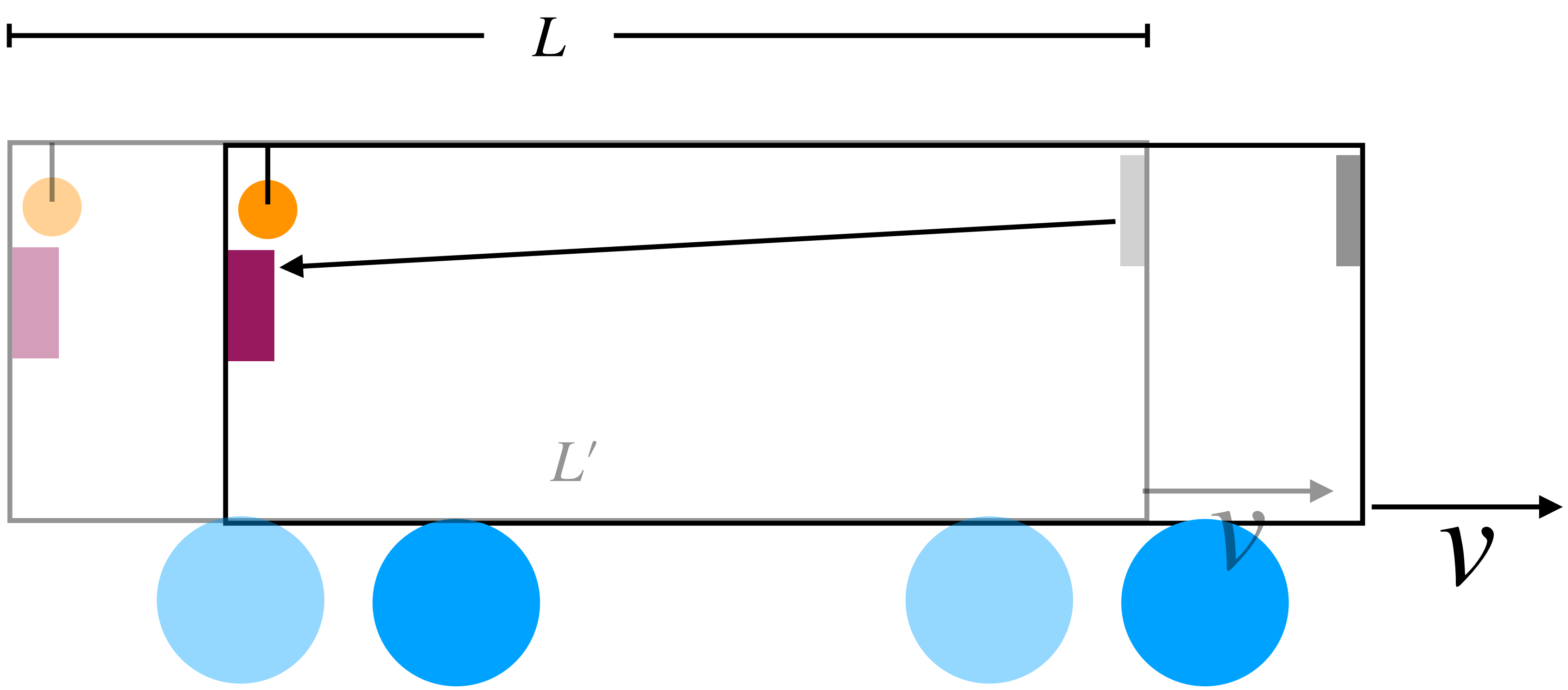
The backwards trip takes a time Δt_2, so

$$\Delta t_2 = \frac{L - v\Delta t_2}{c}$$

$$\Delta t_2 \left(1 + \frac{v}{c}\right) = \frac{L}{c}$$

$$\Delta t_2 = \frac{L}{c}\left(1 + \frac{v}{c}\right)^{-1}$$



(3) Length Contraction (Lorentz Contraction): Again we'll use a

(A) On train;

$$\Delta t' = \frac{2L'}{c}$$

(B) On ground; The roundtrip time is

$$\Delta t = \Delta t_1 + \Delta t_2 = \frac{L}{c}\left(1 - \frac{v}{c}\right)^{-1} + \frac{L}{c}\left(1 + \frac{v}{c}\right)^{-1} = \frac{L/c}{\left(1 - \frac{v}{c}\right)} + \frac{L/c}{\left(1 + \frac{v}{c}\right)}$$

$$= \frac{L/c\left(1 + \frac{v}{c}\right)}{\left(1 - \frac{v^2}{c^2}\right)} + \frac{L/c\left(1 - \frac{v}{c}\right)}{\left(1 - \frac{v^2}{c^2}\right)} = \frac{2L/c}{\left(1 - \frac{v^2}{c^2}\right)} = \gamma^2\frac{2L}{c}.$$

We also have $\Delta t = \gamma \Delta t'$, so putting these together $\Delta t' = \frac{1}{\gamma}\Delta t = \frac{2L'}{c}$.

Then

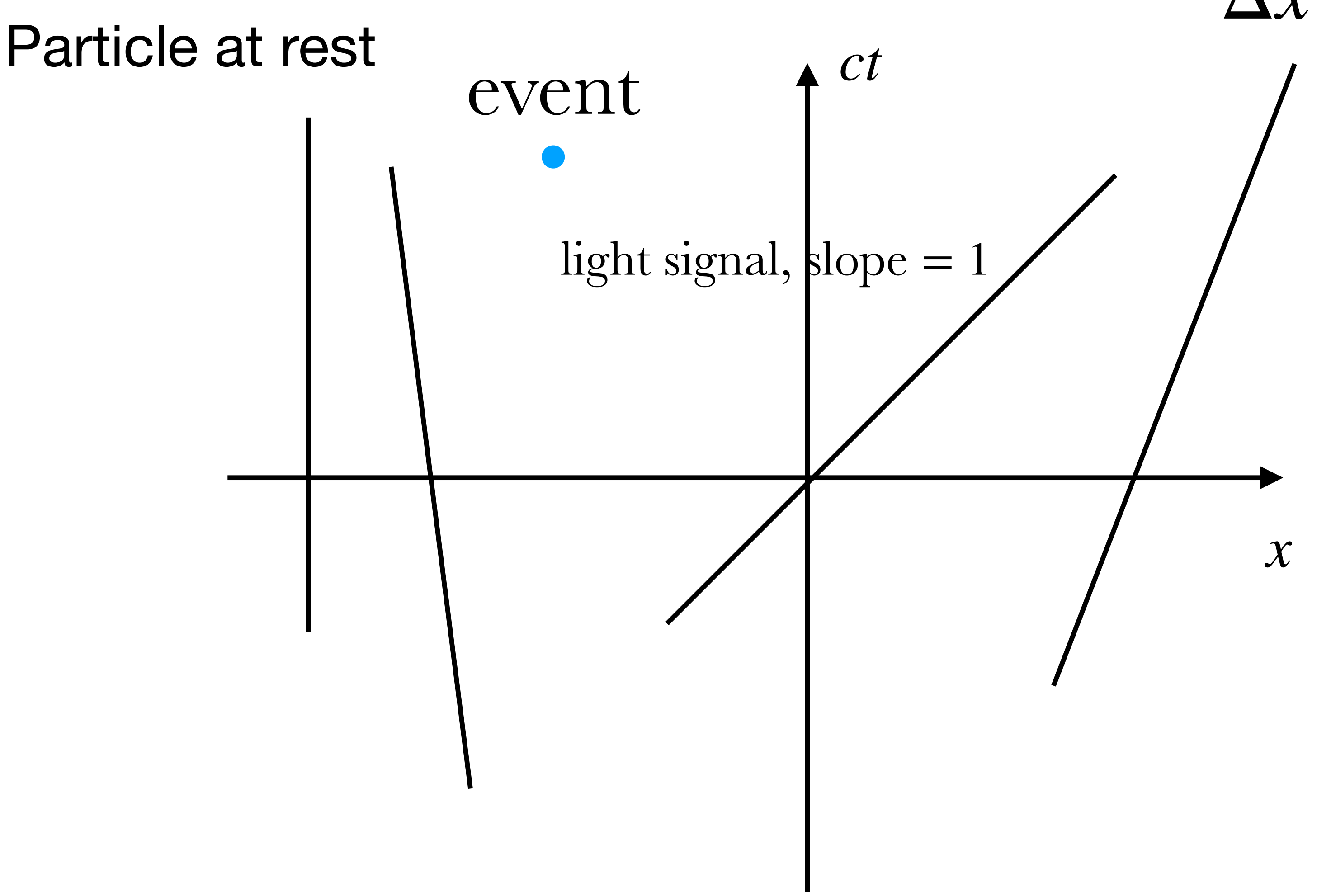
$$\frac{2L'}{c}\gamma = \gamma^2\frac{2L}{c} \implies L' = \gamma L \text{ or } L = \frac{1}{\gamma}L'$$ this is "length contraction".

We won't derive the 4th consequence here, you're doing it on your homework. This is the Einstein velocity addition formula.

III. Spacetime (Minkowski) Diagram

$$\text{slope} = \frac{c\Delta t}{\Delta x} = \frac{c}{\Delta x/\Delta t} = \frac{c}{v}$$

$$\frac{v}{c} = \frac{1}{\text{slope}}$$



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