Today

I. Last Time
II. Office Hour Change Tuesdays, HW Folders, & Lab
III. More on Spacetime (Minkowski) Diagrams
IV. Galilean and Lorentz Transformations

I. We finished deriving three of the four fundamental consequence of Einstein's two postulates.

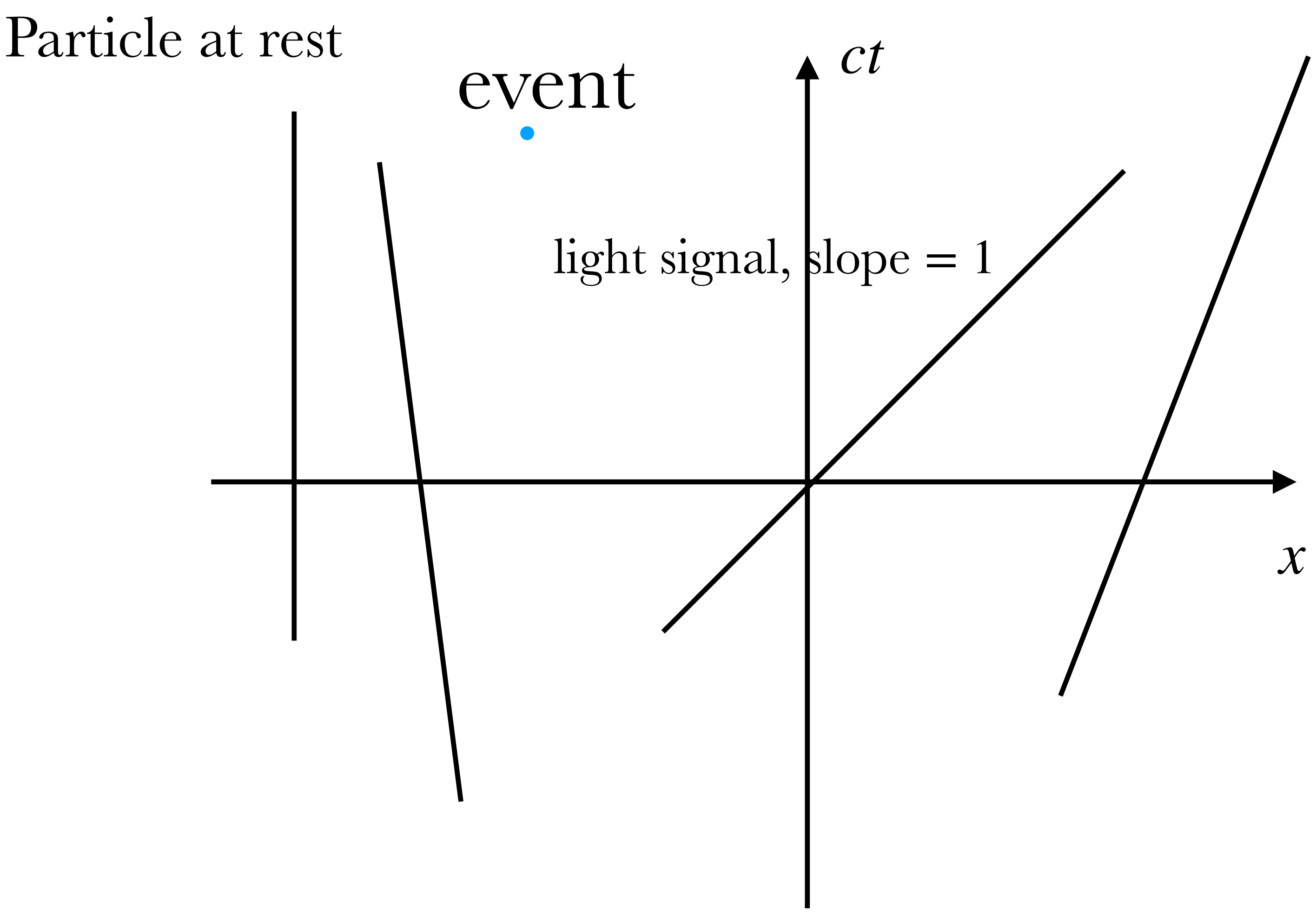
1. Relativity of simultaneity
2. Time dilation: $\Delta t = \gamma \Delta t'$
3. Length (or Lorentz) contraction: $\Delta L = \frac{1}{\gamma} \Delta L'$
4. You are deriving Einstein velocity addition on the homework.

2. and 3. are starting to hint that maybe space and time fit together.

We won't derive the 4th consequence here, you're doing it on your homework. This is the Einstein velocity addition formula.

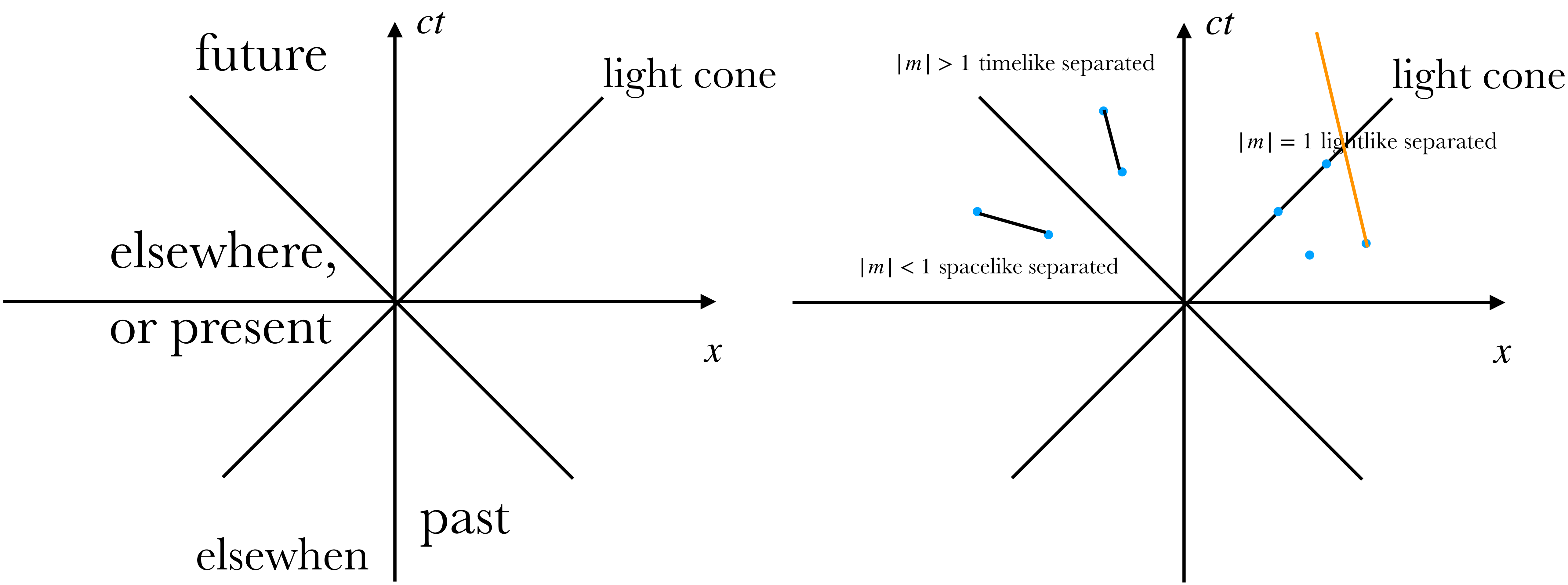
III. More on Spacetime (Minkowski) Diagram

$$\text{slope} = \frac{\text{rise}}{\text{run}} = \frac{c\Delta t}{\Delta x} = \frac{c}{\Delta x/\Delta t} = \frac{c}{v}$$



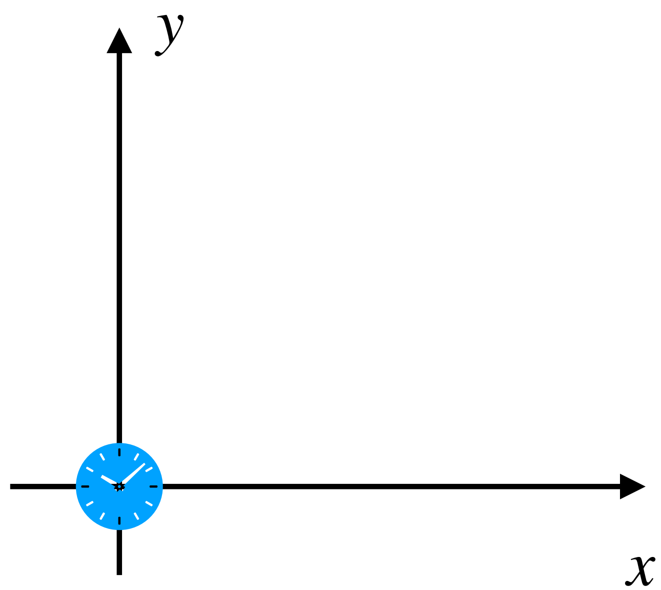
$$\frac{v}{c} = \frac{1}{\text{slope}}$$

III. Spacetime (Minkowski) Diagram

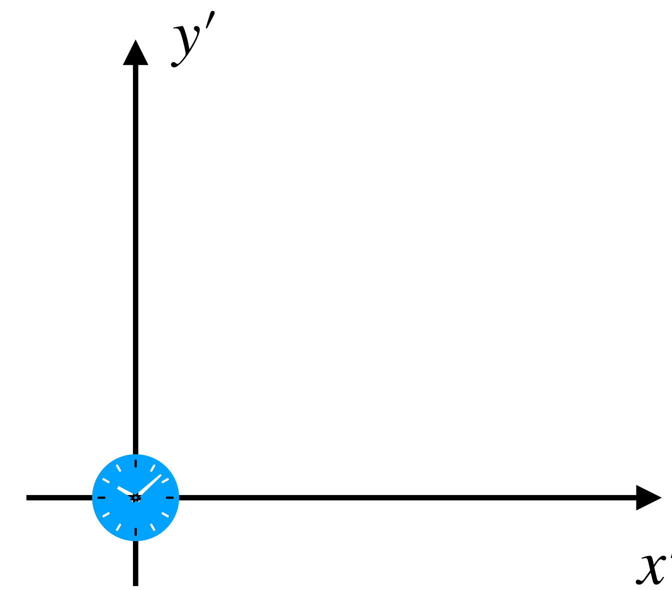


All of the diagrams that I've drawn so far are making reference to to just one reference frame. Ultimately we want the dictionary that connects different frames of reference. This dictionary is called the Lorentz transformation.

IV. Galilean and Lorentz Transformations



Frame S (“rest”, “ground”, “lab”)



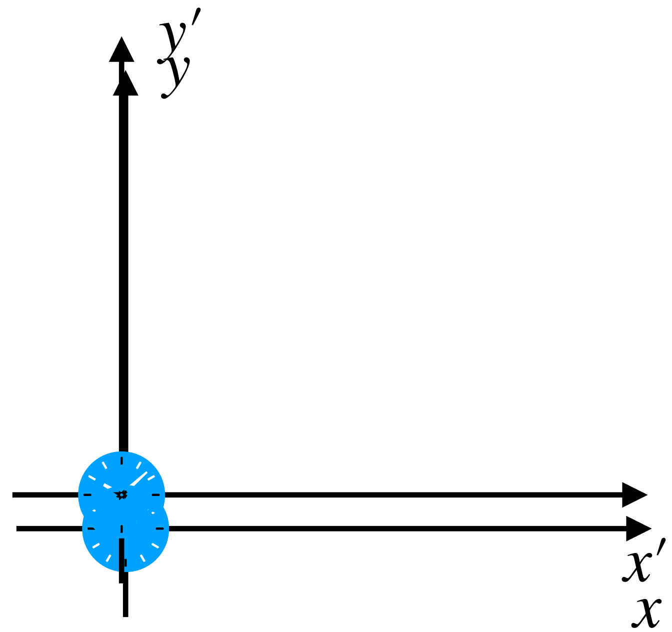
Frame S' (“moving" frame, “train”, “rocket”)

Event: particular spacetime locations (x, y, z) and t (all in S).

Question: What are the coordinates (x', y', z', t') of the SAME EVENT in S'?

Let's assume that frame S' moves at speed v (const.) relative to S, along the x-axis.

IV. Galilean and Lorentz Transformations



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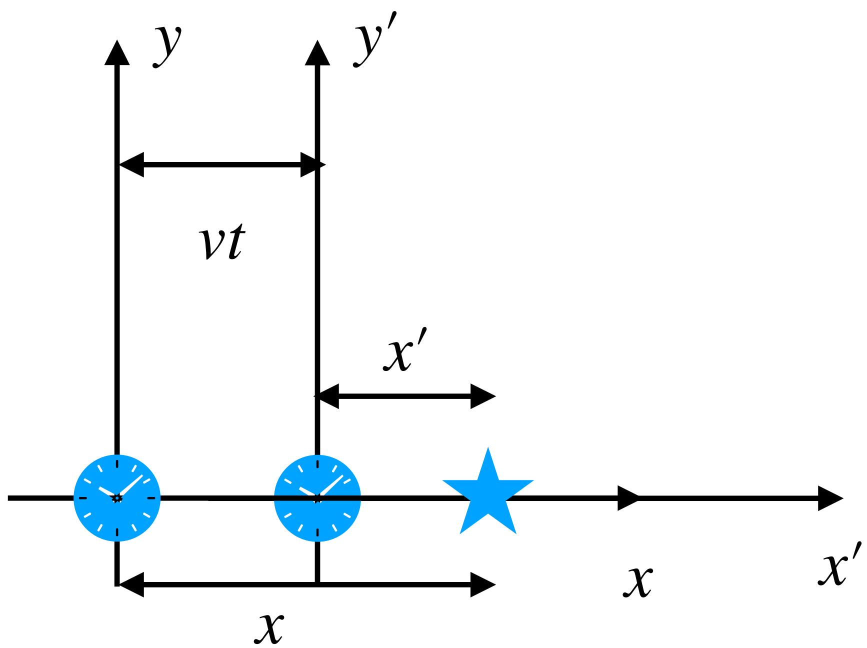
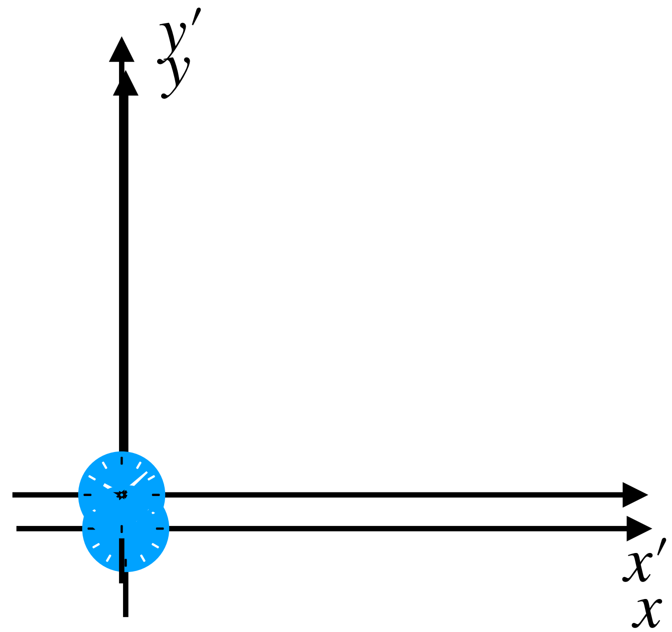


Frame S' (“moving" frame, “train”, “rocket”)

Let's assume that frame S' moves at speed v (const.) relative to S, along the x-axis. At the instant the two origins coincide, we set the two master clocks to zero. After time t, the origin of S' has moved a distance vt.

IV. Galilean and Lorentz Transformations



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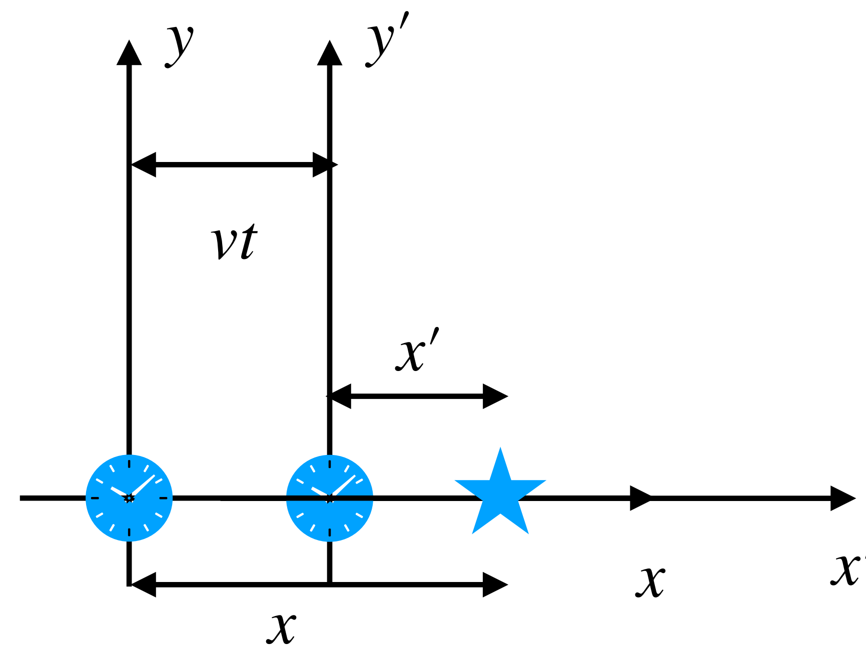


Frame S' (“moving" frame, “train”, “rocket”)

For Galilean transformations we have

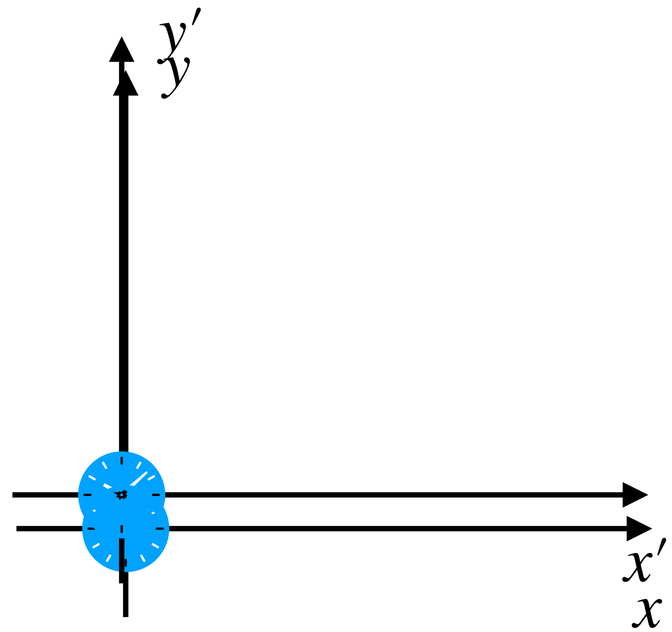
$$x' = x - vt$$

$$y' = y$$

$$z' = z$$

$$t' = t$$

The Lorentz Transformations



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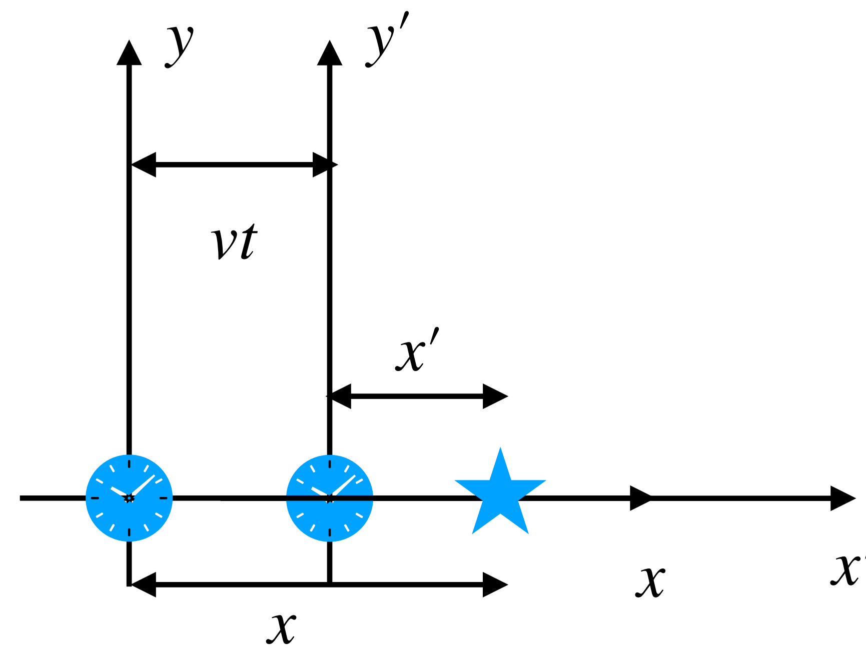


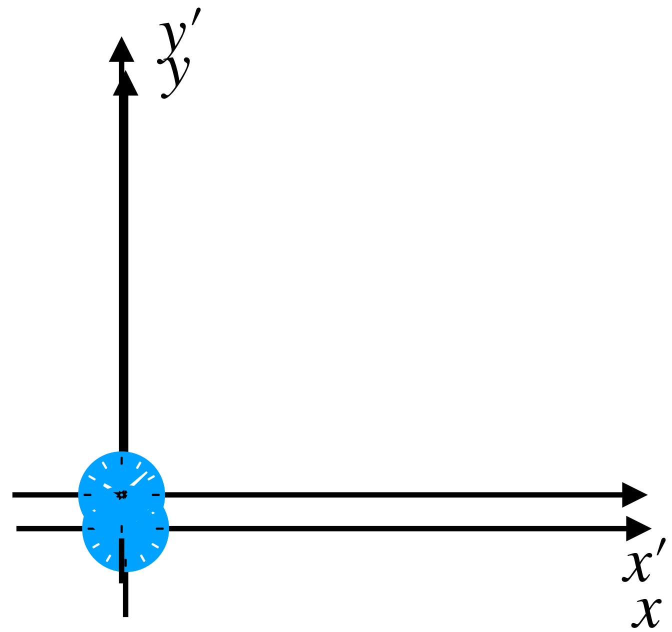
Frame S' (“moving" frame, “train”, “rocket”)

Take perspective of S. The Galilean “x” failed to account for length contraction. Moving objects are shorter than in their rest frame, so

$x' = \gamma(x - vt)$

$y' = y$, $z' = z$. Not done yet(!), need time $t' = \gamma(t - \frac{v}{c^2}x)$

The Lorentz Transformations



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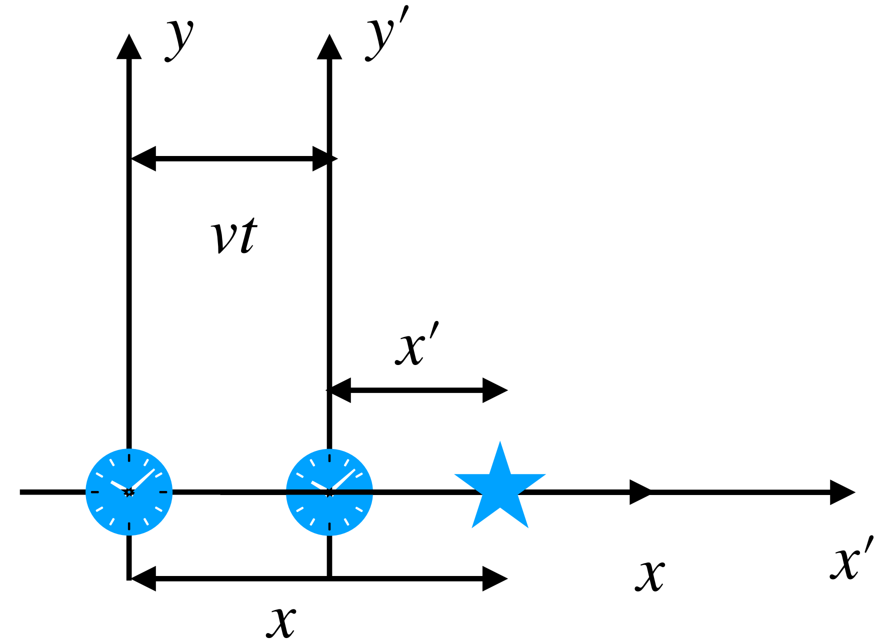

Frame S' (“moving" frame, “train”, “rocket”)

Let’s derive this: $t' = \gamma(t - \frac{v}{c^2}x)$. Inverse transformation of position: $x = \gamma(x' + vt') = \gamma[\gamma(x - vt) + vt'] = \gamma^2 x - \gamma^2 vt + \gamma vt'$.

$\gamma vt' = (1 - \gamma^2)x + \gamma^2 vt$

$$t' = \frac{(1 - \gamma^2)}{\gamma v}x + \gamma t$$

The Lorentz Transformations

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$$\gamma vt' = (1 - \gamma^2)x + \gamma^2 vt$$

$$t' = \frac{(1 - \gamma^2)}{\gamma v}x + \gamma t$$

Focusing on the term in front of x

$$\frac{1}{v}\frac{1 - \gamma^2}{\gamma} = \frac{\gamma}{v}\frac{1 - \gamma^2}{\gamma^2} = \frac{\gamma}{v}\left(\frac{1}{\gamma^2} - 1\right) = \frac{\gamma}{v}\left(1 - \frac{v^2}{c^2} - 1\right) = -\frac{\gamma v}{c^2}$$

Plugging this in gives

$$t' = \gamma t - \frac{\gamma v}{c^2}x = \gamma\left(t - \frac{v}{c^2}x\right).$$

In summary, we have two results

$$x' = \gamma(x - vt) \quad \text{and.} \quad t' = \gamma\left(t - \frac{v}{c^2}x\right)$$

Time meets
2-2:45
Spencer
Woochan
Skylar
Ollie
Talia
Antonio

Space
2:40-3:20
Bobby
Jade
Rose
Thomas
Grace
Quincy