Today

I. Last Time
II. Logistical Questions?
III. Lorentz Transformations: Velocity Addition & the Spacetime Invariant

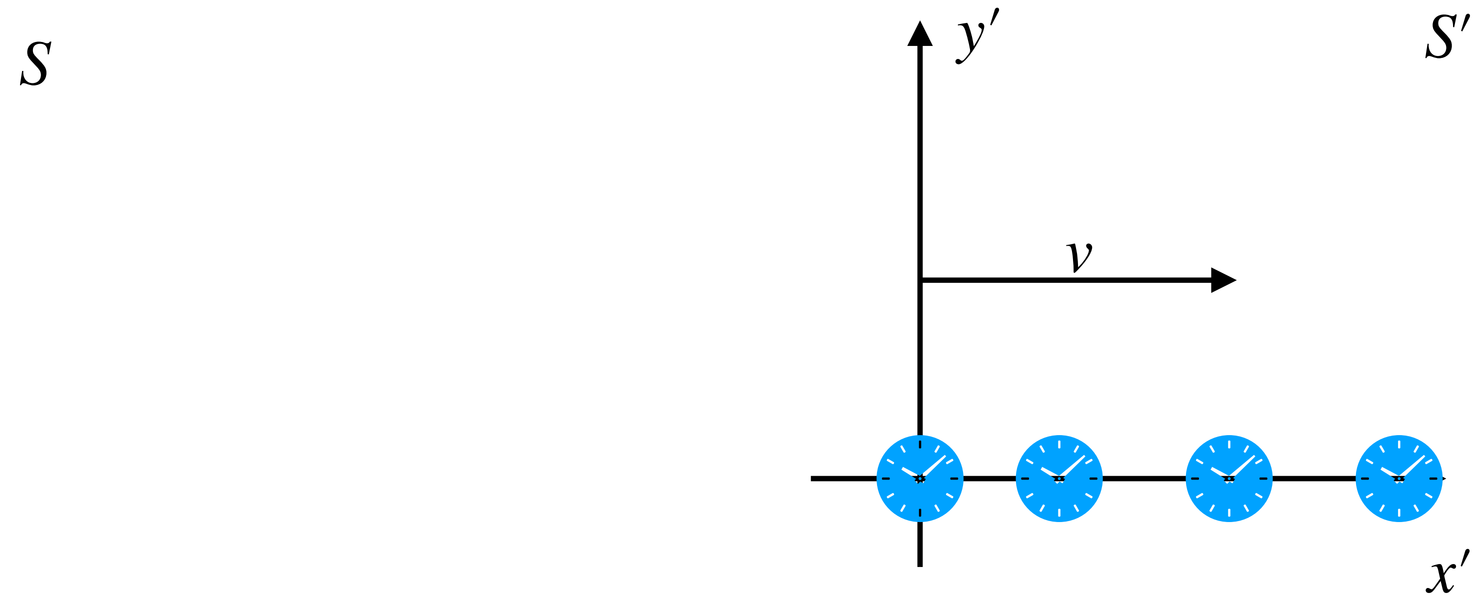
I. We finished talking Minkowski or Spacetime diagrams. Introduced coordinates, (ct, x, y, z).

Derived the dictionary from the coordinates of one frame S, to another one say S'. This dictionary is called a Lorentz transformation:

$$x' = \gamma(x - \frac{v}{c}ct)$$

$$ct' = \gamma(ct - \frac{v}{c}x),\ y' = y, \text{ and } z' = z$$

III. Examples of Lorentz transformations



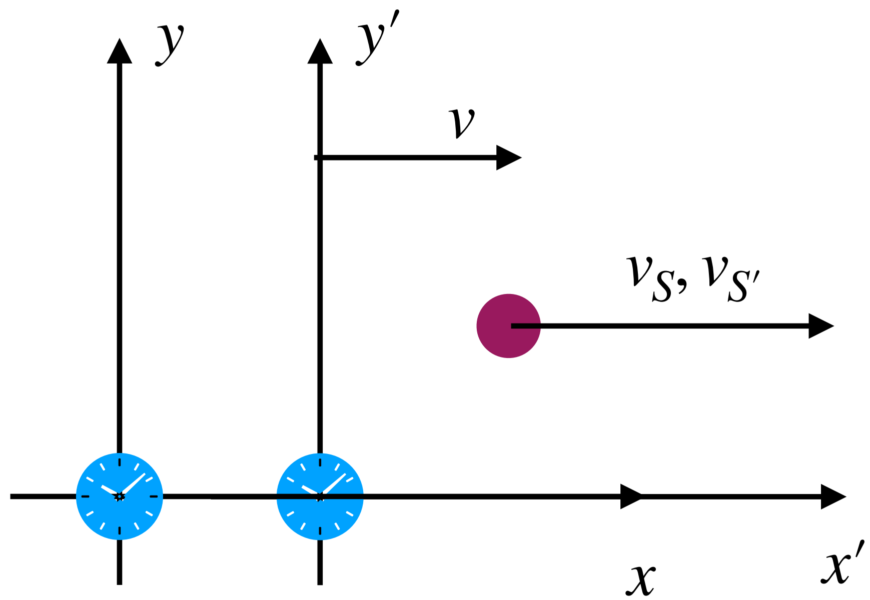
Ex: Non-synchronization of clocks. Say $t = 0$ in S; and our observer in S looks at the S' clocks:

$$t' = \gamma(t - \frac{v}{c^2}x) = -\gamma\frac{v}{c^2}x.$$

The observer in S claims that one reason that they disagree on simultaneity is that the S' observer's clocks are all out of synch.

III. Examples of Lorentz transformations

Example 2:



In S:

$$\Delta x = x_2 - x_1$$
$$\Delta t = t_2 - t_1$$

then $\Longrightarrow v_S = \frac{\Delta x}{\Delta t}.$

In S':

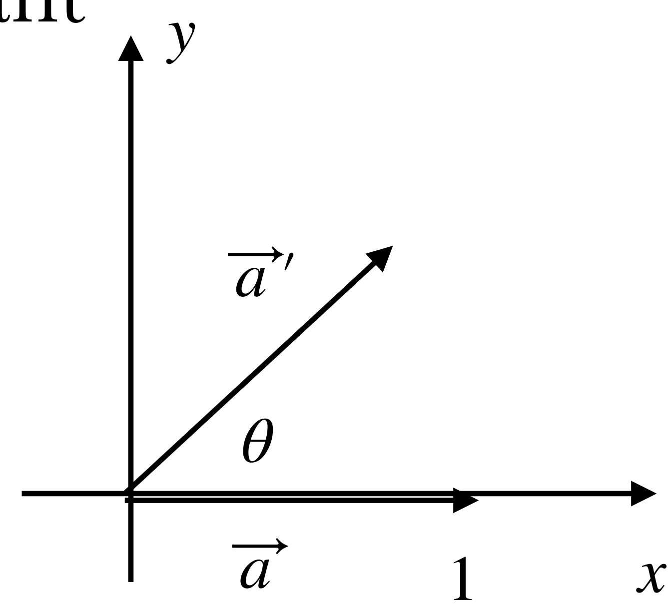
$$\Delta x' = x_2' - x_1' = \gamma(\Delta x - v\Delta t)$$

$$\Delta t' = t_2' - t_1' = \gamma(t_2 - \frac{v}{c^2}x_2) - \gamma(t_1 - \frac{v}{c^2}x_1) = \gamma(\Delta t - \frac{v}{c^2}\Delta x)$$

$$v_{S'} = \frac{\Delta x'}{\Delta t'} = \frac{\gamma(\Delta x - v\Delta t)}{\gamma(\Delta t - \frac{v}{c^2}\Delta x)} = \frac{\Delta t}{\Delta t}\frac{\frac{\Delta x}{\Delta t} - v}{1 - \frac{v}{c^2}\frac{\Delta x}{\Delta t}} = \frac{v_S - v}{1 - \frac{v v_S}{c^2}}!$$

III. Examples of Lorentz transformations

The Spacetime invariant



$\vec{a} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$, then the vector rotated through an angle is

$\vec{a}' = \begin{pmatrix} \cos\theta \\ \sin\theta \end{pmatrix}$. Does this rotation leave anything about our vector alone? Yes. The length. To calculate this we do

$$|\vec{a}|^2 = a_x^2 + a_y^2 = |\vec{a}'|^2 = a_x'^2 + a_y'^2$$

III. Examples of Lorentz transformations

Question: Is there something like length in the spacetime context? That is, is there something that's invariant under Lorentz transformations?

One suggestion, let's try squaring all coordinates and adding 'em up:

$\Delta x^2 + \Delta y^2 + \Delta z^2 + c^2 \Delta t^2 \neq \Delta x'^2 + \Delta y'^2 + \Delta z'^2 + c^2 \Delta t'^2$

The first guess doesn’t work! Instead, time behaves differently

$-c^2 \Delta t^2 + \Delta x^2 + \Delta y^2 + \Delta z^2 = -c^2 \Delta t'^2 + \Delta x'^2 + \Delta y'^2 + \Delta z'^2$

Different resources have different sign conventions: either, like ours, $- + + +$ or $+ - - -$.

Time meets
2-2:45
Spencer
Woochan
Skylar
Ollie
Talia
Antonio

Space
2:40-3:20
Bobby
Jade
Rose
Thomas
Grace
Quincy