Today

I. Small Logistics: Lab Help & More Homework Help
II. Last Time
III. Lorentz Transformations: the Invariant Spacetime Interval
IV. Four Vectors

I. Yanpei Deng will help in the lab. She's available M-W from 6-7pm in Brody lab.

Antu Antu will providing homework support.

Lab Reports now due: 5pm.

II. We discussed the invariance of the spacetime interval

$$-c^2\Delta t^2 + \Delta x^2 = -c^2\Delta t'^2 + \Delta x'^2.$$

Lorentz transformations: $x' = \gamma(x - \frac{v}{c}(ct))$ and $t' = \gamma(t - \frac{v}{c^2}x)$

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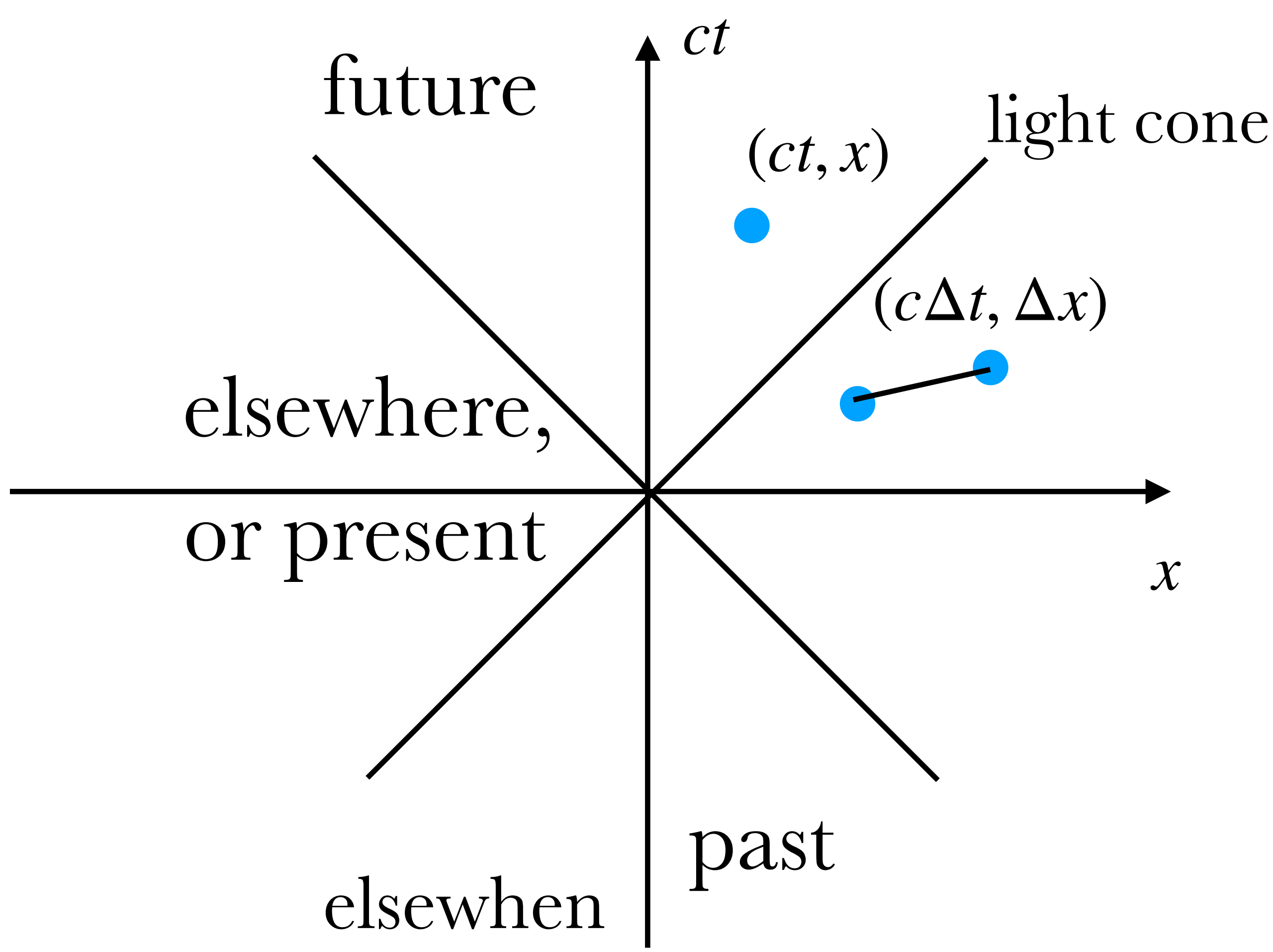
Or

$$ct' = \gamma(ct - \frac{v}{c}x).$$

We also derived that clocks synchronized in one frame appear unsynched in another and we derived velocity addition from Lorentz transformations.

II. We've discussed both spacetime events and spacetime intervals:



$$\Delta x' = \gamma(\Delta x - \frac{v}{c} c\Delta t)$$

$$c\Delta t' = \gamma(\Delta t - \frac{v}{c}\Delta x)$$

Let's check the invariance of the spacetime interval:

$$-c^2\Delta t'^2 + \Delta x'^2 = \gamma^2\left[-c^2(c\Delta t - \frac{v}{c}\Delta x)^2 + (\Delta x - \frac{v}{c}c\Delta t)^2\right]$$

III. We've discussed both spacetime events and spacetime intervals:

Let's check the invariance of the spacetime interval:

$$-c^2\Delta t'^2 + \Delta x'^2 = \gamma^2\left[-(c\Delta t - \frac{v}{c}\Delta x)^2 + (\Delta x - \frac{v}{c}c\Delta t)^2\right]$$

$$= \gamma^2\left[-(c^2\Delta t^2 + \frac{v^2}{c^2}\Delta x^2 - 2v\Delta t\Delta x) + (\Delta x^2 + v^2\Delta t^2 - 2v\Delta x\Delta t)\right]$$

$$= \gamma^2\left[(v^2 - c^2)\Delta t^2 + (1 - \frac{v^2}{c^2})\Delta x^2\right]$$

$$= \gamma^2\left[-\frac{c^2}{\gamma^2}\Delta t^2 + \frac{1}{\gamma^2}\Delta x^2\right] = -c^2\Delta t^2 + \Delta x^2$$

$$\Delta x' = \gamma(\Delta x - \frac{v}{c}c\Delta t)$$

$$c\Delta t' = \gamma(\Delta t - \frac{v}{c}\Delta x)$$

The framing of a calculation in terms these invariant calculations is much, much, much faster.

IV. Four Vectors

Let's introduce a tidier notation:

$x^0 \equiv ct,\, x^1 = x,\, x^2 = y,\, x^3 = z.$

This is called the 4-vector notation. In particular,

$x^\mu = (x^0, x^1, x^2, x^3)$, $\mu \in \{0,1,2,3\}$. The "position-time" 4-vector.

[In Newtonian mechanics we would have written $\overrightarrow{a} = (a^1, a^2, a^3)$.]

$$\Delta x' = \gamma(\Delta x - \frac{v}{c} c\Delta t)$$

$$c\Delta t' = \gamma(\Delta t - \frac{v}{c}\Delta x)$$

Einstein summation notation:

$$\sum_{\mu=0}^{3} x^\mu x_\mu \equiv x^\mu x_\mu.$$

Greek indices are all four values 0,1,2,3 and roman indices are only the three 1,2,3.

Let's also introduce the shorthand $\beta \equiv \frac{v}{c}$. This all cleans up our Lorentz transformations: $(x^0)' = \gamma(x^0 - \beta x^1)$, $(x^1)' = \gamma(x^1 - \beta x^0)$

IV. Four Vectors

What distinguishes a 4-vector from any other physical quantity?

A "4-vector" is any collection of 4 #'s

$a^\mu = (a^0, a^1, a^2, a^3)$

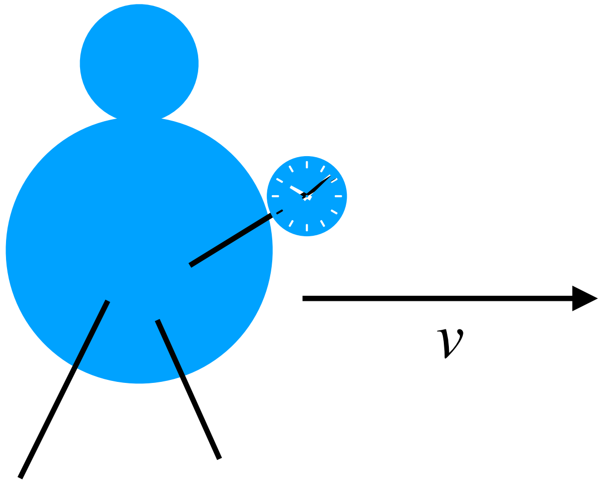
That transforms in the same way that x^μ does when you go from frame S to frame S'. Then,

$(a^0)' = \gamma(a^0 - \beta a^1)$,

$(a^1)' = \gamma(a^1 - \beta a^0)$,

$(a^2)' = a^2$,

$(a^3)' = a^3$.



Let's build up some examples.

Proper time: There's more than one time out there: "ordinary time" ("coordinate" or "lab" time), the time on the lab's wall clock —t. A particle zooming through the lab measures "proper time" on its wrist watch.

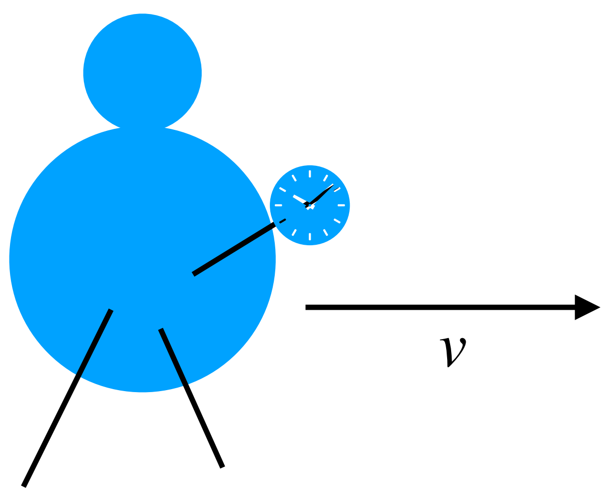
IV. Four Vectors

Let's build up some examples.

<u>Proper time</u>: There's more than one time out there: "ordinary time" ("coordinate" or "lab" time), the time on the lab's wall clock —t. A particle zooming through the lab measures "proper time" on its wrist watch, call it τ.

We know that these two are related by

$$dt = \gamma d\tau.$$



<u>Proper velocity:</u> Ordinary velocity is

$$v = \frac{\Delta x}{\Delta t},$$

with both measured in the lab. Our new notion, of proper velocity, is

$$\eta \equiv \frac{\Delta x}{\Delta \tau} = \frac{dx}{d\tau}!$$