Today

I. Some Logistics: Lab Help & Homework Help Times
II. Last Time
III. Examples of Four Vectors
IV. Energy & Momentum in Special Relativity

I. Yanpei Deng will help in the lab. She's available MTuW from 6-7pm in Brody lab. Gonna try to move all three to 7-8pm.

Antu Antu will be providing homework support. Possible hours are: Tu 8-9pm, Th10:30-11:30am, Th 8-9pm.

(Th btwn 9am-2pm, and 7-10pm, Tu 8-11pm)
Hw due: Thursday 10pm
Lab Reports now due: Saturday 5pm.

II. We discussed 4-vectors:

$x^0 = ct, x^1 = x, x^2 = y, x^3 = z$

$x^\mu = (ct, x, y, z), \quad \mu \in \{0,1,2,3\}$ labels a single event!

This gave us a very succinct and symmetrical way of writing Lorentz transformations

$(x^0)' = \gamma(x^0 - \beta x^1)$

$(x^1)' = \gamma(x^1 - \beta x^0)$

$(x^2)' = x^2, (x^3)' = x^3, \quad \text{here } \beta \equiv \frac{v}{c}.$

Defined a general 4-vector a^μ, which is a vector that has components that transform under a Lorentz transformation in exactly the same way as the components of the spacetime 4-vector. We also call Lorentz transformations "boosts".

III. We also introduced "proper time": particle's wrist watch time, call it τ. Lab clocks measure lab time t, and so these two times differ by

$$dt = \gamma d\tau.$$

This led us to introduce a new notion of velocity, proper velocity

$$\eta = \frac{\Delta x}{\Delta \tau}.$$

Lab measurement

Particle measurement

Let's consider the chain rule

$$\eta_x = \frac{dx}{d\tau} = \frac{dx}{dt}\frac{dt}{d\tau} = \gamma \frac{dx}{dt} = \gamma v_x.$$

Again,

$$\eta_y = \frac{dy}{d\tau} = \frac{dy}{dt}\frac{dt}{d\tau} = \gamma v_y.$$

Then we have a 4-vector:

$$\eta^\mu = \frac{dx^\mu}{d\tau} = (c\frac{dt}{d\tau}, \gamma v_x, \gamma v_y, \gamma v_z) = \gamma(c, v_x, v_y, v_z).$$

IV. Relativistic Energy and Momentum

Classically, what is momentum?

$$\overrightarrow{p} = m\overrightarrow{v}.$$

In relativity we could define momentum as

$$\overrightarrow{p} = m\overrightarrow{v} \qquad \text{or} \qquad \overrightarrow{p} = m\overrightarrow{\eta}.$$

Which is the right one to use? When we were mechanics it was *essential* that momentum was conserved! In order to have conservation of momentum in any reference frame we *must* use the second definition.

<u>Define:</u> Relativistic momentum is:

$$\overrightarrow{p} = m\overrightarrow{\eta} = \gamma m\overrightarrow{v} = \frac{m\overrightarrow{v}}{\sqrt{1 - \frac{v^2}{c^2}}}.$$

Don't forget $v^2 = \overrightarrow{v} \cdot \overrightarrow{v} = v_x^2 + v_y^2 + v_z^2$.

IV. Define: Relativistic momentum is:

$$\overrightarrow{p} = m\overrightarrow{\eta} = \gamma m\overrightarrow{v} = \frac{m\overrightarrow{v}}{\sqrt{1-\frac{v^2}{c^2}}}.$$

Don't forget $v^2 = \overrightarrow{v} \cdot \overrightarrow{v} = v_x^2 + v_y^2 + v_z^2$.

<u>Define:</u> Relativist energy is the zeroth component of the energy-momentum 4-vector and is defined by

$$E = \gamma mc^2.$$

Note that as a momentum we need to divide by a factor of c,

$$p^0 = \frac{E}{c}.$$

Collecting all these ideas together we have the full 4-vector

$p^\mu = (p^0, p^1, p^2, p^3) = (\frac{E}{c}, p_x, p_y, p_z)$. (Nota Bene: $p_x \equiv \gamma m v_x$)

IV. Define: Relativist energy is the zeroth component of the energy-momentum 4-vector and is defined by

$$E = \gamma mc^2.$$

Note that as a momentum we need to divide by a factor of c,

$$p^0 = \frac{E}{c}.$$

Writing out the relativistic energy we have

$$E = \frac{mc^2}{\sqrt{1-\frac{v^2}{c^2}}} = mc^2\left(1-\frac{v^2}{c^2}\right)^{-\frac{1}{2}} \approx mc^2\left(1-(-1/2)\frac{v^2}{c^2}\right)$$

$$= mc^2 + mc^2\frac{1}{2}\frac{v^2}{c^2} = mc^2 + \frac{1}{2}mv^2 + \cdots.$$