Today

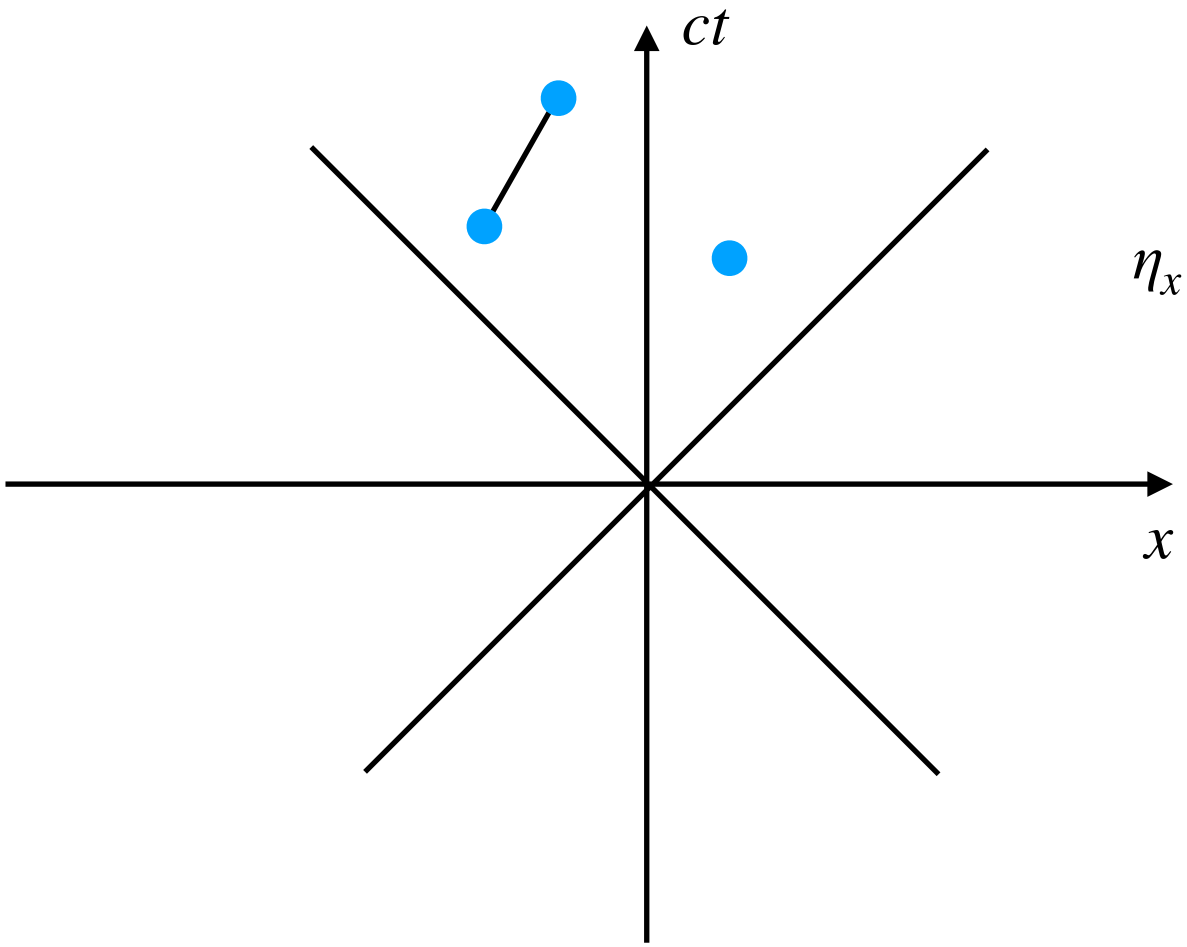
I. Where are we? General questions
II. Last Time
III. Conservation of Relativistic Energy & Momentum
IV. Massless Particles

I. Yanpei Deng will help in the lab. She's available MTuW from 7-8pm in Brody lab.

Antu Antu will be providing homework support. Possible hours are: Tu 8-9pm, Th10:30-11:30am, Th 8-9pm.

Hw due: Thursday 10pm
Lab Reports now due: Saturday 5pm.

I.



$$\eta_x = \lim_{\Delta\tau \to 0} \frac{\Delta x}{\Delta \tau} = \frac{dx}{d\tau}$$

$$\eta^\mu = \gamma(c, v_x, v_y, v_z)$$

II. Last time we discussed relativistic momentum

$$\overrightarrow{p} = \gamma m \overrightarrow{v},$$

and relativistic energy,

$$E = \gamma mc^2.$$

R is the "rest energy"

By calculation we found: $E \approx mc^2 + \frac{1}{2}mv^2 + \cdots = R + K.E. + \cdots$

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We collected these into an "Energy-momentum" 4-vector:

$$p^\mu = \gamma(mc, mv_x, mv_y, mv_z) = \gamma(mc, m\overrightarrow{v}) = (\frac{E}{c}, \overrightarrow{p}).$$

III. Conservation Law:

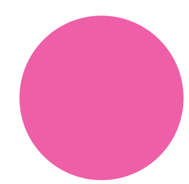
In any collision process (including decays) relativistic energy & momentum are conserved.

Example: Take two relativistic particles of equal mass m with equal and opposite speeds $3/5c$ and let them collide head-on:

III.

before

Assume they stick together after the collision:



after

Question: What's the mass of the composite lump?

To answer this we use relativistic conservation of energy:

(before) (after)

$$\gamma mc^2 + \gamma mc^2 = Mc^2.$$

Let's find γ,

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{1}{\sqrt{1 - \left(\frac{3}{5}\right)^2}} = \frac{1}{\sqrt{\frac{16}{25}}} = \frac{5}{4}$$

III.

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Then,

$$2m\frac{5}{4} = M \implies M = 2.5m$$

In relativity, kinetic energy can be converted into mass.

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III.

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We also have $\pi^0 \to e^- + e^+$, and the mass of the pion is more than twice the mass of the electron. The extra mass is converted into the kinetic energy of the electron/positron pair.

Def: An elastic collision is on in which K.E. is conserved (in practice, same particles out as in).

IV. Massless particles

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Relativity: There is a loophole! If $v = c$, then

$$E = \gamma mc^2 = \frac{0}{0},\ p = \gamma mv = \frac{0}{0}?$$

In analogy with the spacetime invariant,

$$-\frac{E^2}{c^2} + p^2 = -\frac{\gamma^2 m^2 c^4}{c^2} + \gamma^2 m^2 v^2 = m^2\gamma^2(-c^2 + v^2) = m^2\frac{-c^2 + v^2}{1 - \frac{v^2}{c^2}} = -m^2c^2$$

$$m^2\frac{-c^2 + v^2}{1 - \frac{v^2}{c^2}} = m^2\frac{c^2}{c^2}\frac{-c^2 + v^2}{1 - \frac{v^2}{c^2}} = m^2c^2\frac{-c^2 + v^2}{c^2 - v^2} = -m^2c^2.$$

Then, for a massless particle we have that, whatever E and p are, they are related by

$$-E^2/c^2 + p^2 = 0 \implies E^2/c^2 = p^2 \implies E = pc.$$

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This argument is hard to swallow, but we measure them in the lab!

Some examples are:

photons (particle of light)

gluons (particle of the strong force)

graviton (particle of gravity, completely hypothetical)