Today

I. Last Time
II. Extended Example of Relativistic Energy-Momentum Conservation

I. Yanpei Deng will help in the lab. She's available MTuW from 7-8pm in Brody lab.

Antu Antu will be providing homework support. Possible hours are: Tu 8-9pm, Th10:30-11:30am, Th 8-9pm.

Hw due: Thursday 10pm
Lab Reports now due: Saturday 5pm.
First exam is a take-home exam, it will be posted this Friday and be due the following Thursday at 10pm.

I. Last Time

Talked about massless particles and how they are possible in special relativity. In general,

$$E^2 - p^2c^2 = E^2 - \vec{p} \cdot \vec{p}c^2 = m^2c^4,$$

and for a massless particle this becomes

$$E = pc = |\vec{p}|c = \sqrt{p_x^2 + p_y^2 + p_z^2}c = \sqrt{\vec{p} \cdot \vec{p}}\, c.$$

In S.R. we have conservation of relativistic energy and momentum! We can write this in terms a 4-vector equation

$$p^{\mu}_{\text{tot,i}} = p^{\mu}_{\text{tot,f}} \qquad \mu = 0,1,2,3$$

gives all 4 equations. Recall,

$$p^{\mu} = (\frac{E}{c}, p_x, p_y, p_z).$$

Definitions: $E = \gamma mc^2$, $\vec{p} = \gamma m \vec{v}$.

II.

Energy-momentum 4-vector. So far we've thought of γ as a function of v:

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}.$$

Recall $v = \eta/\gamma$,

$$\gamma = \frac{1}{\sqrt{1 - \frac{1}{\gamma^2}\frac{\eta^2}{c^2}}}, \implies \gamma^2 = \frac{1}{1 - \frac{1}{\gamma^2}\frac{\eta^2}{c^2}} \implies \gamma^2\left(1 - \frac{1}{\gamma^2}\frac{\eta^2}{c^2}\right) = 1$$

$$\implies \gamma^2 - \frac{\eta^2}{c^2} = 1 \implies \gamma = \sqrt{1 + \frac{\eta^2}{c^2}}$$

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Back in the 60's and 70's people thought neutrinos might be massless, since then we've learned that they are not. However, they are very low mass. It's an approximation, but for the rest of this course you can treat them as massless (unless I tell you not to).

μ^-: muon, like a heavy electron $m_\mu = 105.7\dfrac{\text{MeV}}{c^2}$

ν_μ: muon neutrino, "little neutral one"

$m_{\nu_\mu} \sim \dfrac{1}{10}\text{ev}/c^2 = 1.78 \times 10^{-37} kg \approx 0.$

Example: $\pi^- \to \mu^- + \overline{\nu_\mu}$, here the overbar indicates

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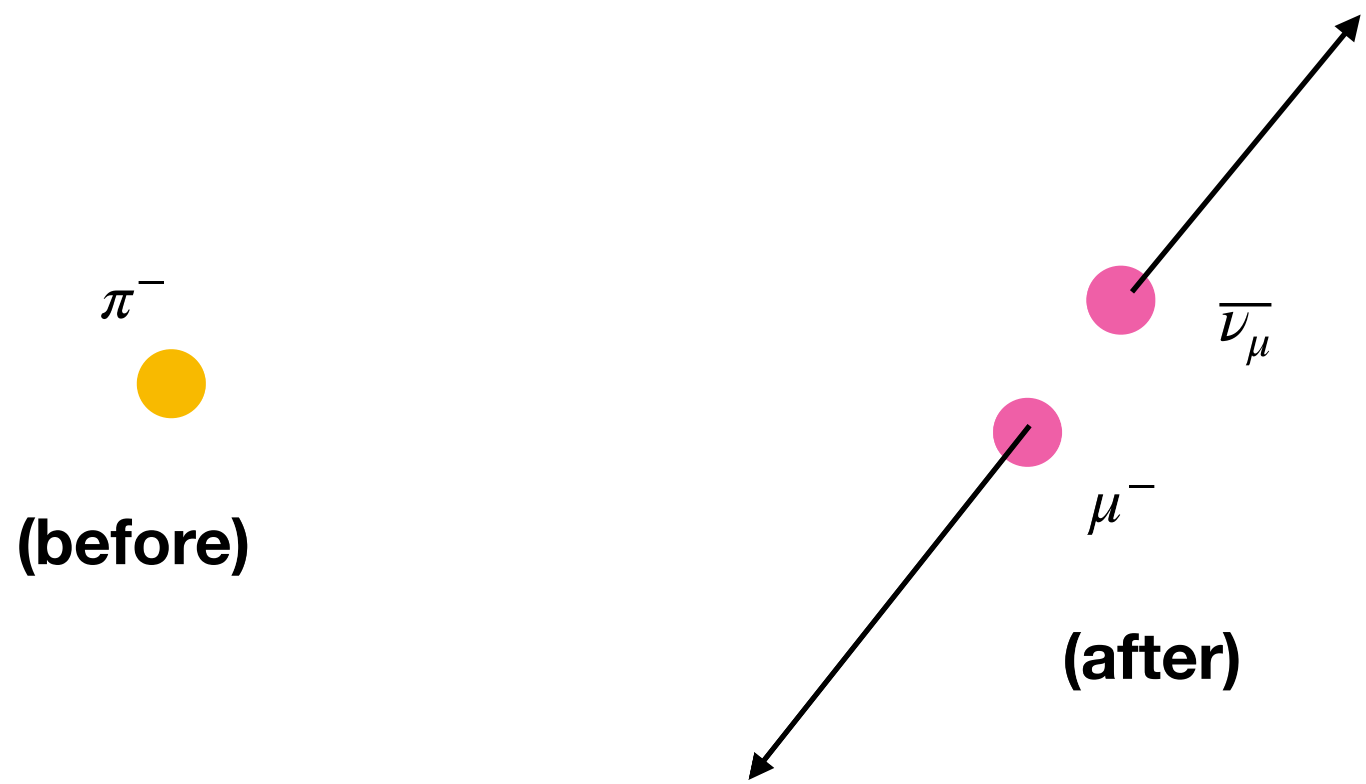

What's the energy of the muon? Conservation of energy says that:

$$m_\pi c^2 = E_\mu + E_\nu = E_\mu + |\overrightarrow{p}_\nu|c.$$

Conservation of momentum says that

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Notice both from the picture and from this second equation $|\vec{p}_\nu| = |\vec{p}_\mu|$. Then we know that

$$m_\pi c^2 = E_\mu + |\vec{p}_\mu| c.$$

The final ingredient is the energy-momentum invariant

$E_\mu^2 - p_\mu^2 c^2 = m_\mu^2 c^4$. ($\mu$ labels muon here)

$$p_\mu^2 c^2 = E_\mu^2 - m_\mu^2 c^4 \implies |\vec{p}_\mu| c = \sqrt{E_\mu^2 - m_\mu^2 c^4}$$

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We rewrite and square

$$(m_\pi c^2 - E_\mu)^2 = m_\pi^2 c^4 - 2m_\pi c^2 E_\mu + E_\mu^2 = E_\mu^2 - m_\mu^2 c^4,$$

$$m_\pi^2 c^4 + m_\mu^2 c^4 = 2m_\pi c^2 E_\mu,$$

$$\boxed{E_\mu = \frac{m_\pi^2 c^2 + m_\mu^2 c^2}{2m_\pi}.}$$