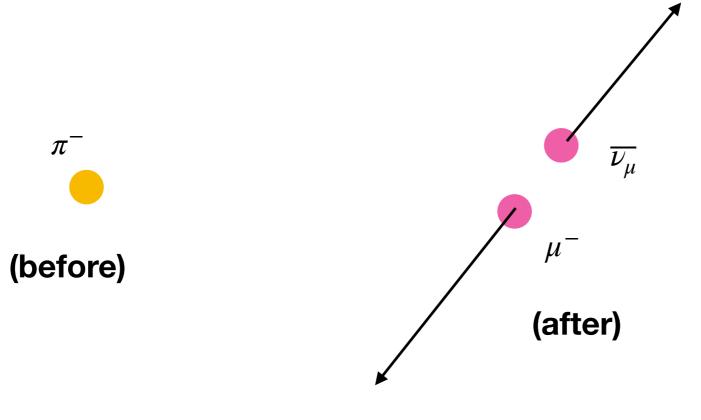
## <u>Today</u> I. Last Time

- II. Wrap up of Extended Example of Relativistic Energy-Momentum Conservation: Suggestions for Relativistic particle problems
- III. Last Comments on Relativity
- IV. Complex Numbers
  - I. Yanpei Deng will help in the lab. She's available MTuW from 7-8pm in Brody lab.
  - Antu Antu will be providing homework support. Possible hours are: Tu 8-9pm, Th10:30-11:30am, Th 8-9pm.
  - Hw due: Thursday 10pm
  - Lab Reports now due: Saturday 5pm.
  - First exam is a take-home exam, it will be posted this Friday and be due the following Thursday at 10pm.

## I. Last Time

Last time we were discussing the decay of a pion into a muon and anti-muon neutrino.



We found that 
$$E_{\mu} = \frac{m_{\pi}^2 c^2 + m_{\mu}^2 c^2}{2m_{\pi}}$$

Can we also find the muon's momentum? What about its speed? II. Let's do the momentum:

$$|\overrightarrow{p}_{\mu}|c = \sqrt{E_{\mu}^2 - m_{\mu}^2 c^4}.$$

I. We found that  $E_{\mu} = \frac{m_{\pi}^2 c^2 + m_{\mu}^2 c^2}{2m_{\pi}}$ .

Can we also find the muon's momentum? What about its speed? II. Let's do the momentum:

$$\vec{p}_{\mu} | c = \sqrt{E_{\mu}^2 - m_{\mu}^2 c^4} = \sqrt{\left(\frac{m_{\pi}^2 c^2 + m_{\mu}^2 c^2}{2m_{\pi}}\right)^2 - m_{\mu}^2 c^4}$$
$$= \sqrt{\frac{m_{\pi}^4 c^4 + m_{\mu}^4 c^4 + 2m_{\pi}^2 m_{\mu}^2 c^4 - 4m_{\pi}^2 m_{\mu}^2 c^4}{4m_{\pi}^2}}$$
$$= \sqrt{\left(\frac{m_{\pi}^2 c^2 - m_{\mu}^2 c^2}{2m_{\pi}}\right)^2} = \left(\frac{m_{\pi}^2 c^2 - m_{\mu}^2 c^2}{2m_{\pi}}\right)$$

**Suggestion 1**: To get the *E* of a particle given its  $\overrightarrow{p}$  (or vice versa) use the invariant.

II. Let's also find the velocity of the muon.

Recall  $E = \gamma mc^2$  and  $\overrightarrow{p} = \gamma m \overrightarrow{v}$ , notice that if we take the ratio we have

$$\frac{\overrightarrow{p}}{E} = \frac{\gamma m \overrightarrow{v}}{\gamma m c^2} = \frac{\overrightarrow{v}}{c^2} \quad \text{or } \overrightarrow{v} = \frac{\overrightarrow{p} c^2}{E}$$

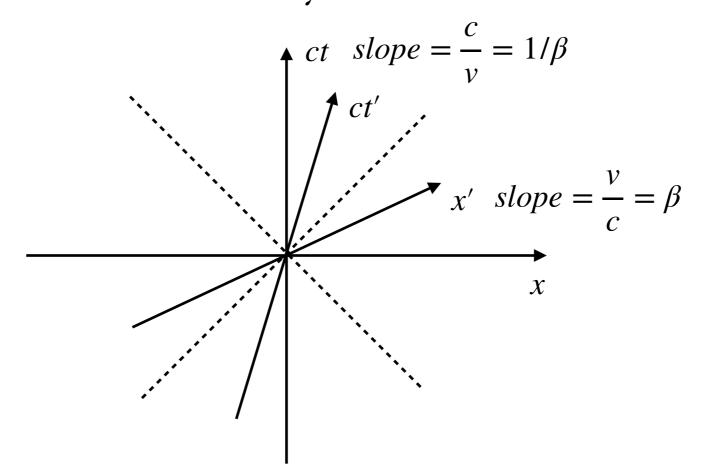
Let's do it for our muon

$$|\vec{v}_{\mu}| = \frac{|\vec{p}_{\mu}|c^{2}}{E_{\mu}} = \frac{\left(\frac{m_{\pi}^{2}c^{3} - m_{\mu}^{2}c^{3}}{2m_{\pi}}\right)}{\frac{m_{\pi}^{2}c^{2} + m_{\mu}^{2}c^{2}}{2m_{\pi}}} = \frac{m_{\pi}^{2} - m_{\mu}^{2}}{m_{\pi}^{2} + m_{\mu}^{2}}c.$$

**Suggestion 1**: To get the *E* of a particle given its  $\overrightarrow{p}$  (or vice versa) use the invariant.

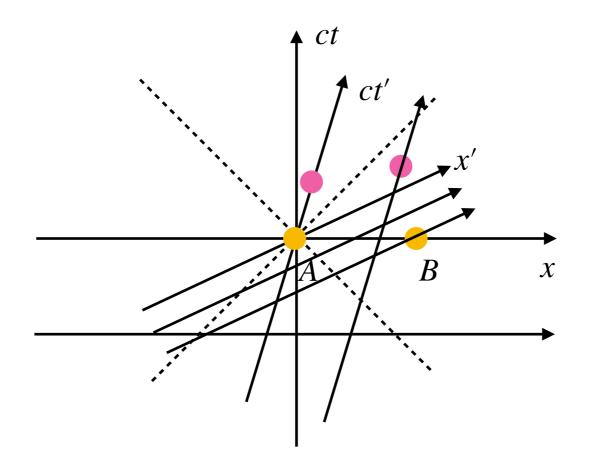
**Suggestion 2**: If you know *E* and  $\overrightarrow{p}$  of a particle and you want  $\overrightarrow{v}$ , then use  $\overrightarrow{v} = \overrightarrow{p}c^2/E$ .

III. Last comments on relativity



This gives us a graphical way to understand the two lights problem with the relativistic train.

## III. Last comments on relativity

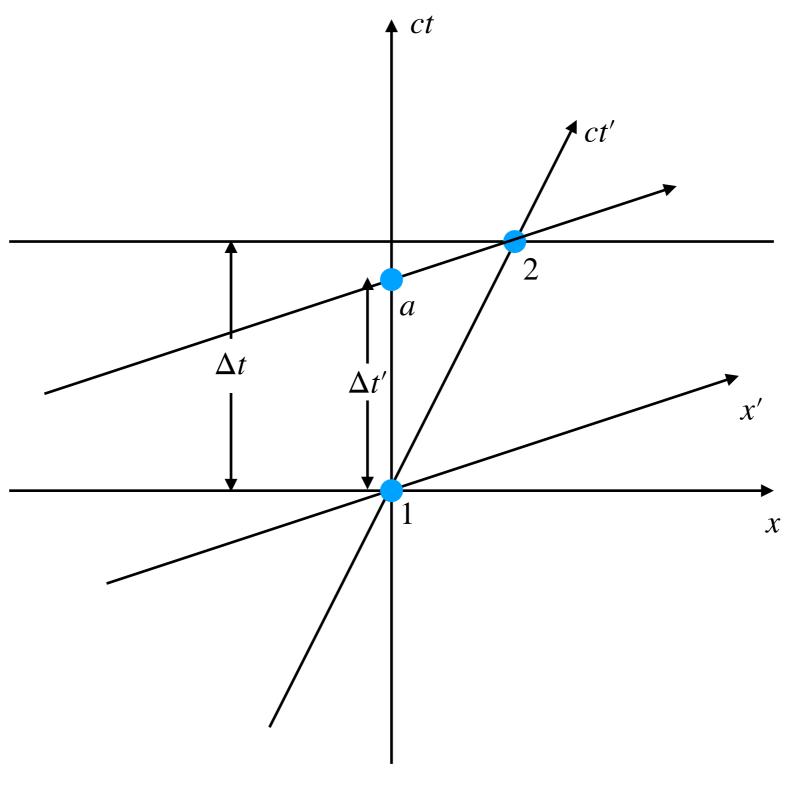


This gives us a graphical way to understand the two lights problem with the relativistic train.

 $\Delta s^2 = -c^2 \Delta t^2 + \Delta x^2 \text{ (note strange minus again)}$ 

To find the coordinates of an event in any frame, we take parallels to the axes, move them to the events and follow them back to the axes.

## III. Time dilation in diagrams:



 $\Delta t = \gamma \Delta t'$