Today

I. Questions about the Exam?
II. Last Time
III. Complex Numbers
IV. Preview Ideas on Waves

I. Yanpei Deng will help in the lab. She's available MTuW from 7-8pm in Brody lab.

Antu Antu will be providing homework support. Possible hours are: Tu 8-9pm, Th10:30-11:30am, Th 8-9pm.

Hw due: Thursday 11:59pm

Lab Reports now due: Saturday 5pm.

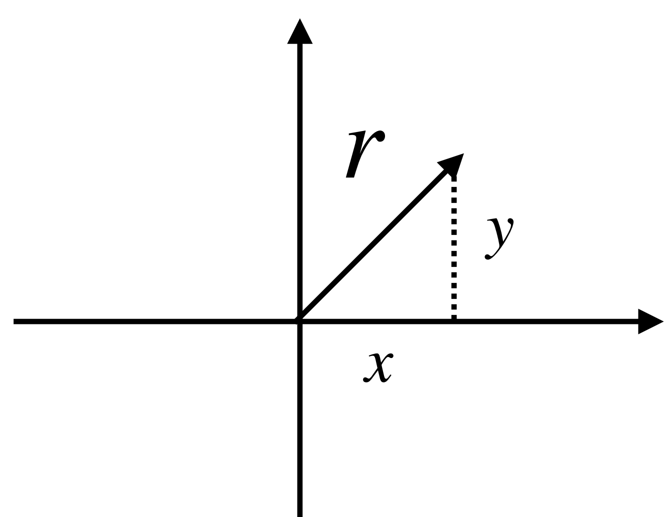
First exam is a take-home exam, it will be posted this Friday and be due the following Thursday at 11:59pm.

I. Last Time

Spacetime diagrams, and in particular, picturing multiple frames on a single diagram. Looked time dilation, relativity of simultaneity.

Another look a 2D spaces is useful and here we'll be thinking about their description in terms of complex numbers. We'll be focused on using them to describe mostly spatial behaviors.

II. In Euclidean geometry we have the Pythagorean theorem:



$r^2 = x^2 + y^2$

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I. Last Time

What a complex number does is to encode to real numbers in a single object. We call this $z = x + iy$, where $i \equiv \sqrt{-1}$.

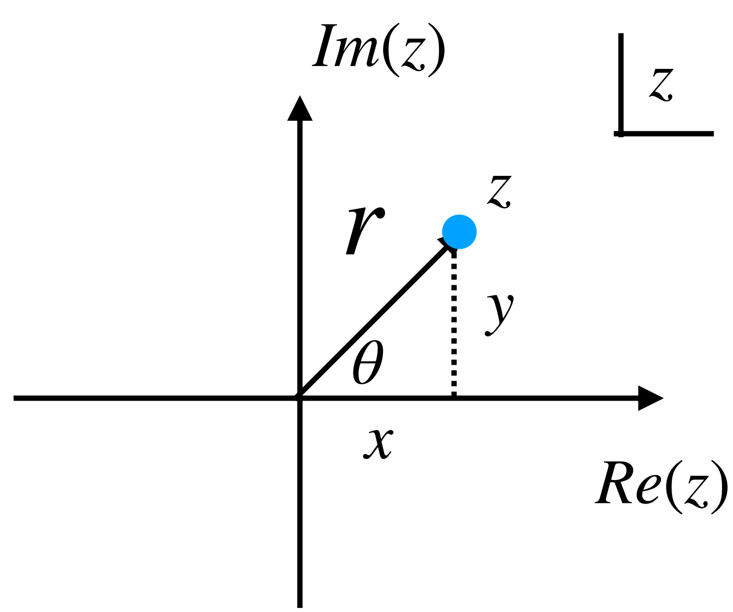
With these numbers we can solve

$$z^2 = -1 \implies z = \pm i.$$

$$r = \sqrt{x^2 + y^2} = \sqrt{r^2\cos^2\theta + r^2\sin^2\theta} = r\sqrt{\cos^2\theta + \sin^2\theta} = r$$

The rules of algebra work exactly the same way for complex numbers. I'm allowed to do things like i^i.

Conventionally we Take r to always be positive.



$x = r\cos\theta \qquad y = r\sin\theta$

$$\begin{aligned} z &= r\cos\theta + ir\sin\theta \\ &= r(\cos\theta + i\sin\theta) \\ &= re^{i\theta} \end{aligned}$$

Then,

$$e^{i\theta} = \cos\theta + i\sin\theta$$

$$e^{i\cdot 0} = e^0 = 1 = \cos(0) + i\sin(0) = 1$$

$$e^{i\cdot\frac{\pi}{2}} = \cos(\frac{\pi}{2}) + i\sin(\frac{\pi}{2}) = i\cdot 1 = i$$

A bit more geometry.

To translate in the horizontal direction we add a real number to the given complex number:

$$z \mapsto z + a$$

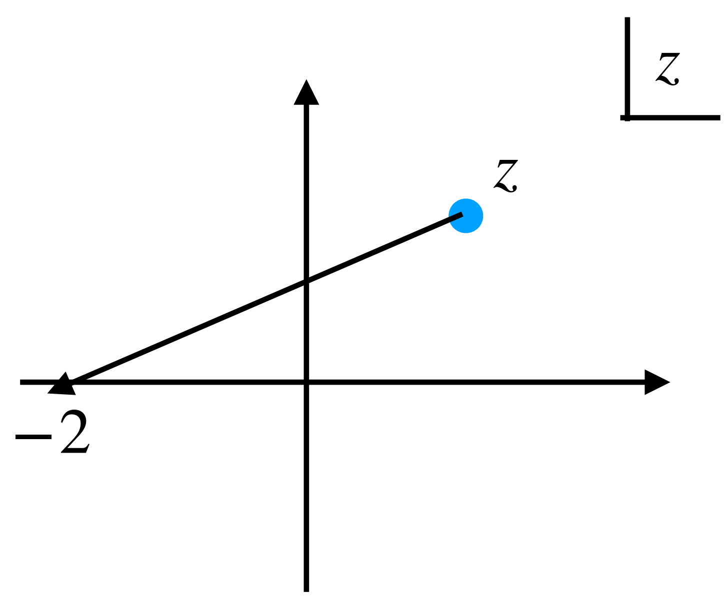
where a is a real number. To translate in the vertical dir we do

$$z \mapsto z + ib$$

where b is real and ib is “purely imaginary”.

What does adding a complex number w to z do? With this technique we can move in any direction by any amount!

To accomplish the translation in the picture we need $w = -z - 2$.



$$x = r\cos\theta \qquad y = r\sin\theta$$

A bit more geometry.

What if I wanted to reflect z across the real axis. I can do this by negating the imaginary part of z. This operation comes up so often that we give it its own name: complex conjugation and denote it by

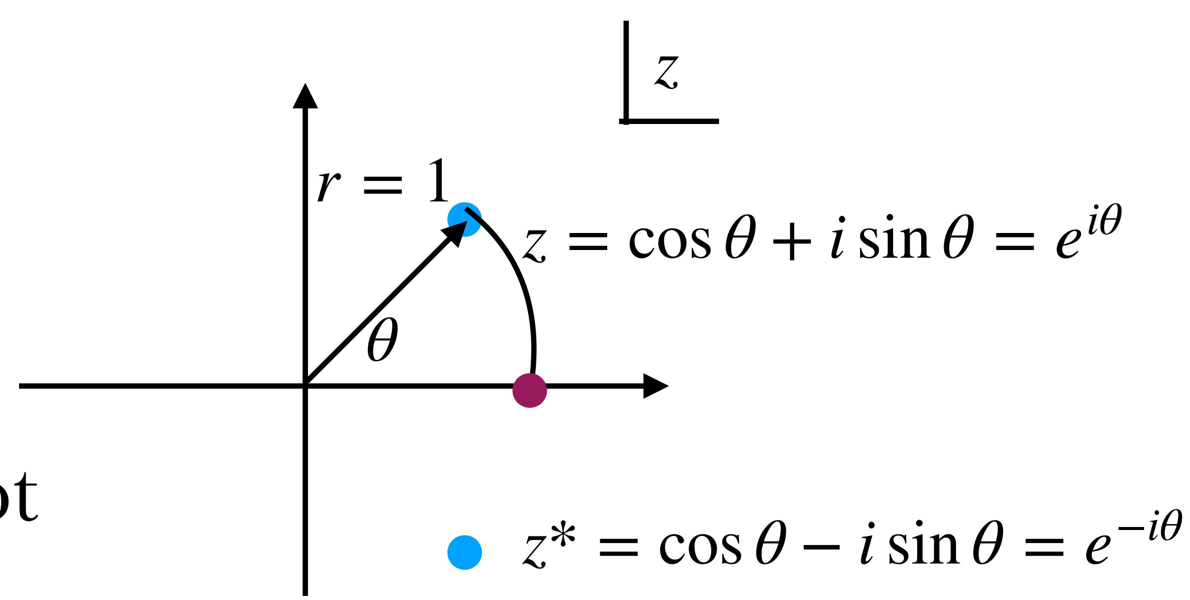
$$z^* = (x + iy)^* \equiv x - iy.$$

It's a theorem that any, arbitrary complicated formula, for a complex number can be complex conjugated simply by replacing every i in it by $-i$.

$$e^{-i\theta} = \frac{1}{e^{i\theta}}$$

$$e^{i\theta} \cdot e^{-i\theta} = 1$$

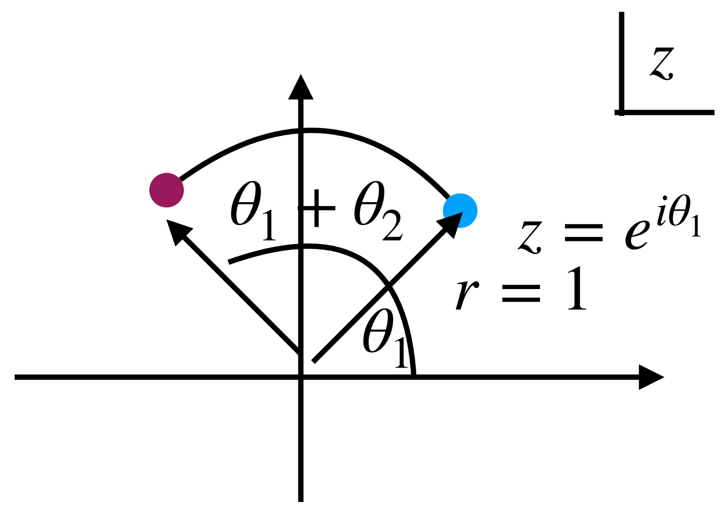
Multiplication by $e^{-i\theta}$ took the blue dot into the purple one.



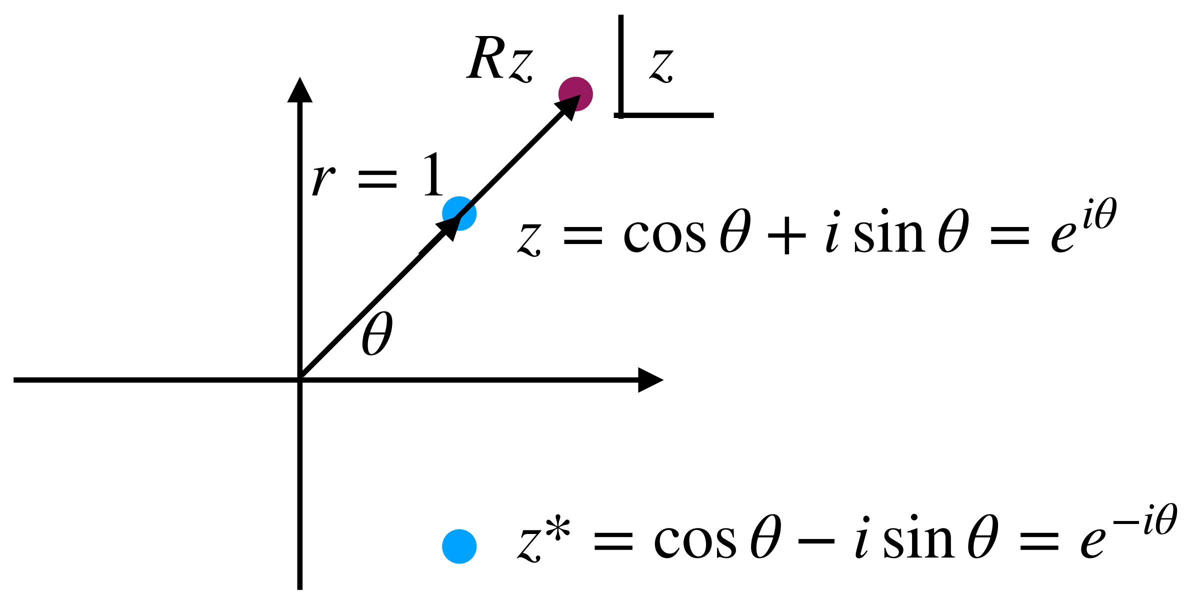
$x = r\cos\theta \qquad y = r\sin\theta$

A bit more geometry.

In general if I start with a complex number $e^{i\theta_1}$ and multiply it by a complex number $e^{i\theta_2}$, I get $e^{i(\theta_1+\theta_2)}$.



If I start with z and multiply by a real number it increases the magnitude: Rz

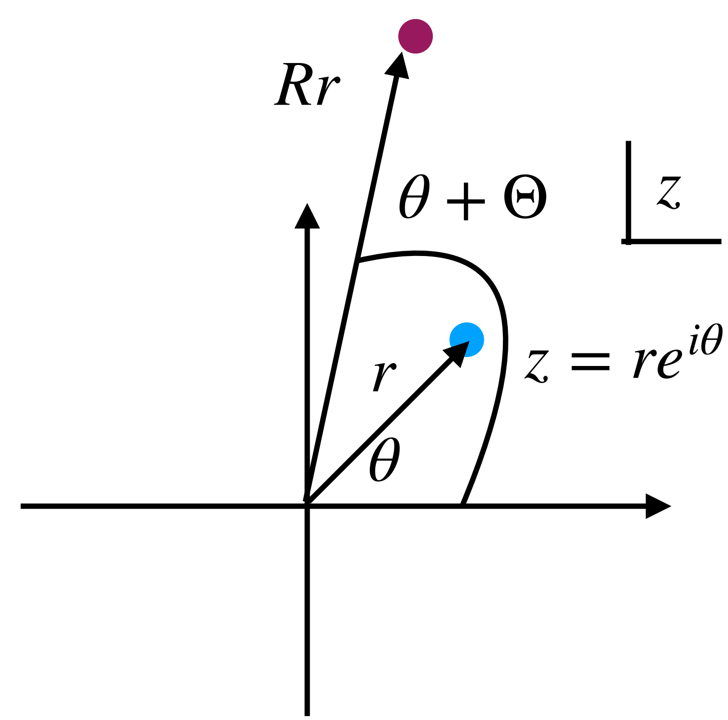


$x = r\cos\theta \qquad y = r\sin\theta$

A bit more geometry.

Given a general z, what happens when I multiply by a general complex number w? We can always decompose w as $w = Re^{i\Theta}$ and once we've done that we have $wz = Rre^{i(\theta+\Theta)}$.

A bit of technology: $z = x + iy$, $z^* = x - iy$, then $Re(z) = \frac{1}{2}(z + z^*)$. $Im(z) = \frac{1}{2i}(z - z^*)$ (N.B.: the imaginary part of a complex number is real!)



We're doing all of this Because of the Euler identity

$e^{i\theta} = \cos(\theta) + i \sin\theta$