Today

I. Questions about the Exam? (Keep in mind that not everyone will have looked at it yet.)
II. Last Time
III. Harmonic Motion
IV. Waves & the Wave Equation

I. No Yanpei Deng this week due to exam (will help in the lab. She's available MTuW from 7-8pm in Brody lab).

Not this week: Antu Antu will be providing homework support. Hours are: Tu 8-9pm, Th10:30-11:30am, Th 8-9pm.

First exam is a take-home exam, it was posted last Friday and is due the following Thursday at 11:59pm.

I. Last Time

Explored complex numbers:

$z^* = (x + iy)^* = x - iy$

$Re(z) = \frac{1}{2}(z + z^*), \quad Im(z) = \frac{1}{2i}(z - z^*)$

$z = re^{i\theta}$, and Euler's identity $\boxed{e^{i\theta} = \cos\theta + i\sin\theta.}$

II. Einstein's 2nd postulate highlights the special nature of light. But, so far we've treated light as a particle. Part of the discovery of light's character came through its wave properties, to which we now turn.

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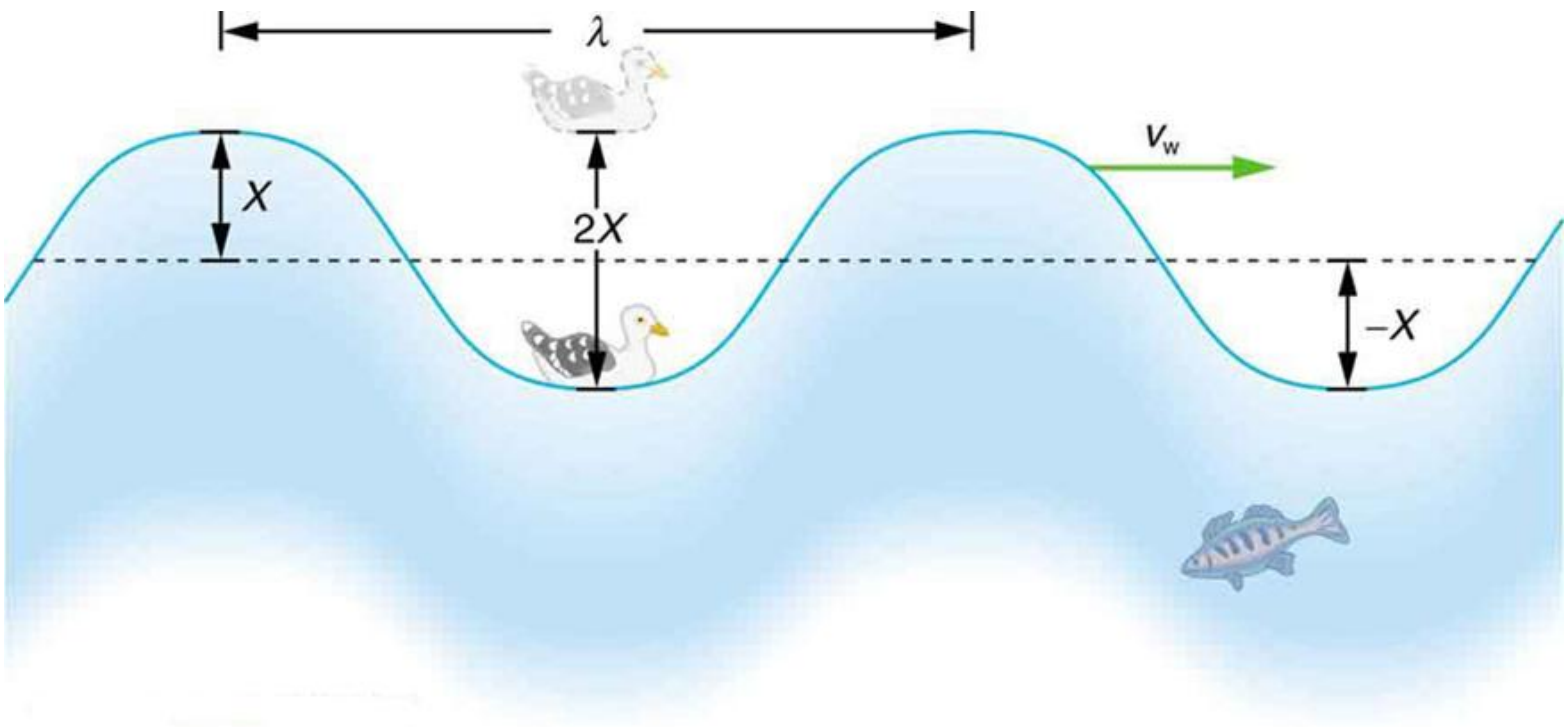
If light is an electromagnetic wave, what is the medium it is waving? This is the question that Michelson and Morley's experiment hinted at answering. (People at the time called this the luminiferous aether.)

II.

How will the duck move? The duck oscillates up and down, up and down. How do we model this motion?

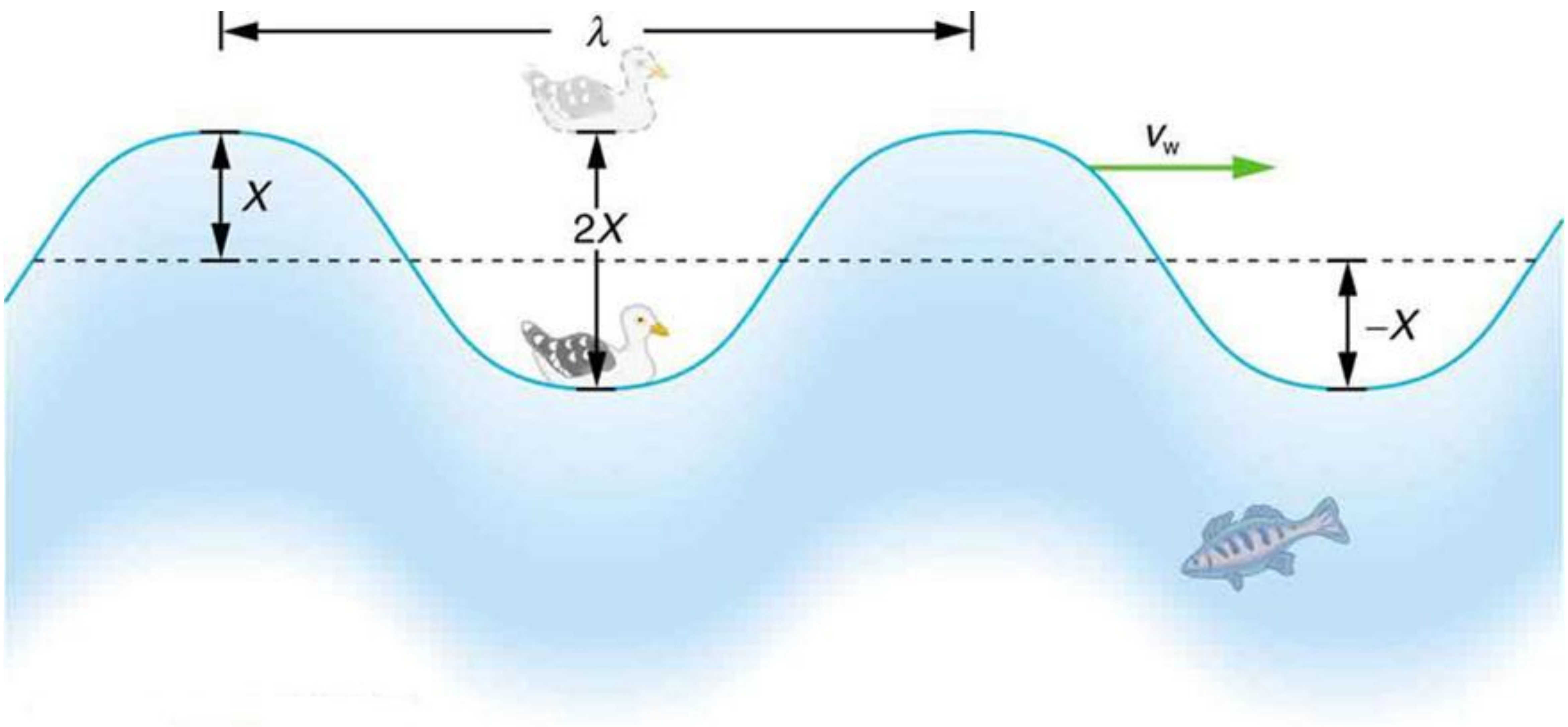
$\psi(t) = A\cos(\omega t + \phi)$, here ω is the "angular frequency" $\omega = 2\pi f$, where f is the standard "frequency" $[f]$=Hertz=1/seconds.



II. According to Newton $F = ma$ and in this case the force is given by a linear restoring force, "Hooke's Law", $F = -k\psi$. Then we have:

$$ma = m\frac{d^2\psi}{dt^2} = -k\psi(t),$$

To get the answer from the previous slide I have to solve a differential equation!

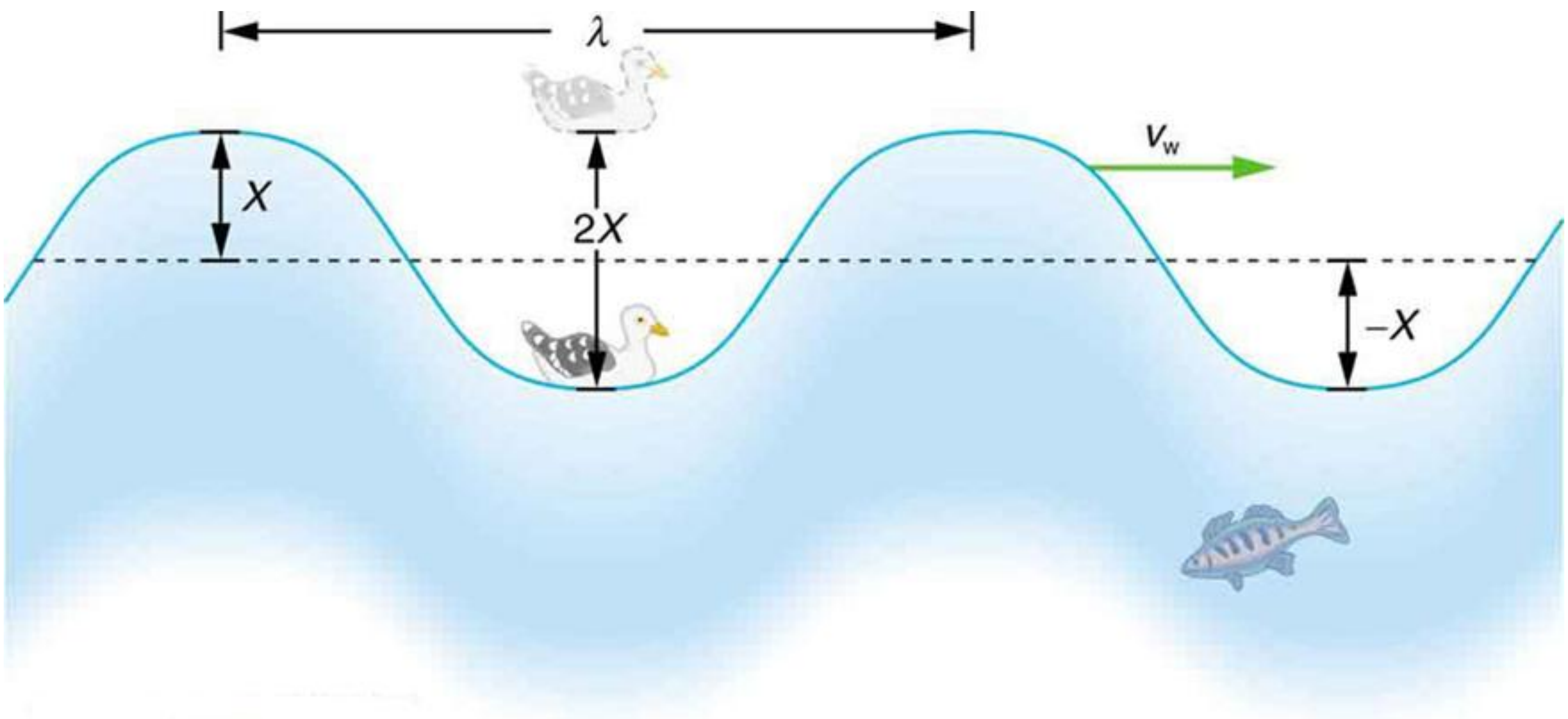


II. Equation of motion of the duck: $ma = m\dfrac{d^2\psi}{dt^2} = -\,k\psi(t)$,

Let's guess $\psi(t) = e^{i\omega t} = \cos(\omega t) + i\sin(\omega t)$, then

$\dfrac{d\psi}{dt} = i\omega e^{i\omega t}$, $\dfrac{d^2\psi}{dt^2} = -\,\omega^2 e^{i\omega t} = -\,\omega^2\psi(t)$ is a solution if and only if

$\omega^2 = k/m$ or, in other words, $\omega = \sqrt{k/m}$.



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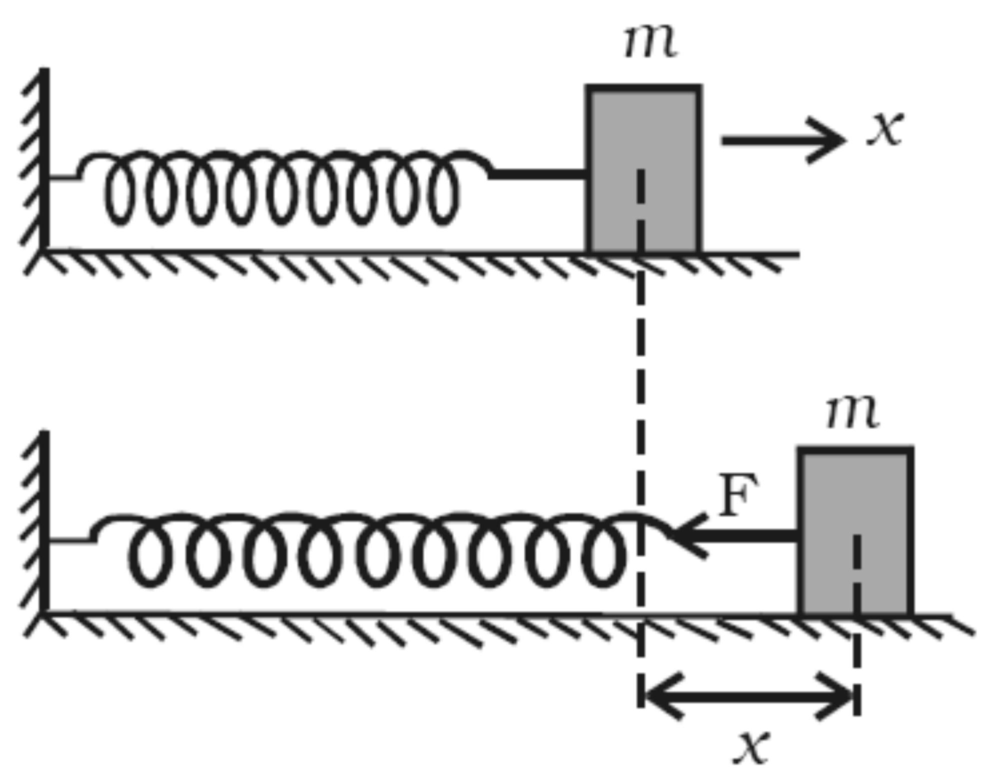
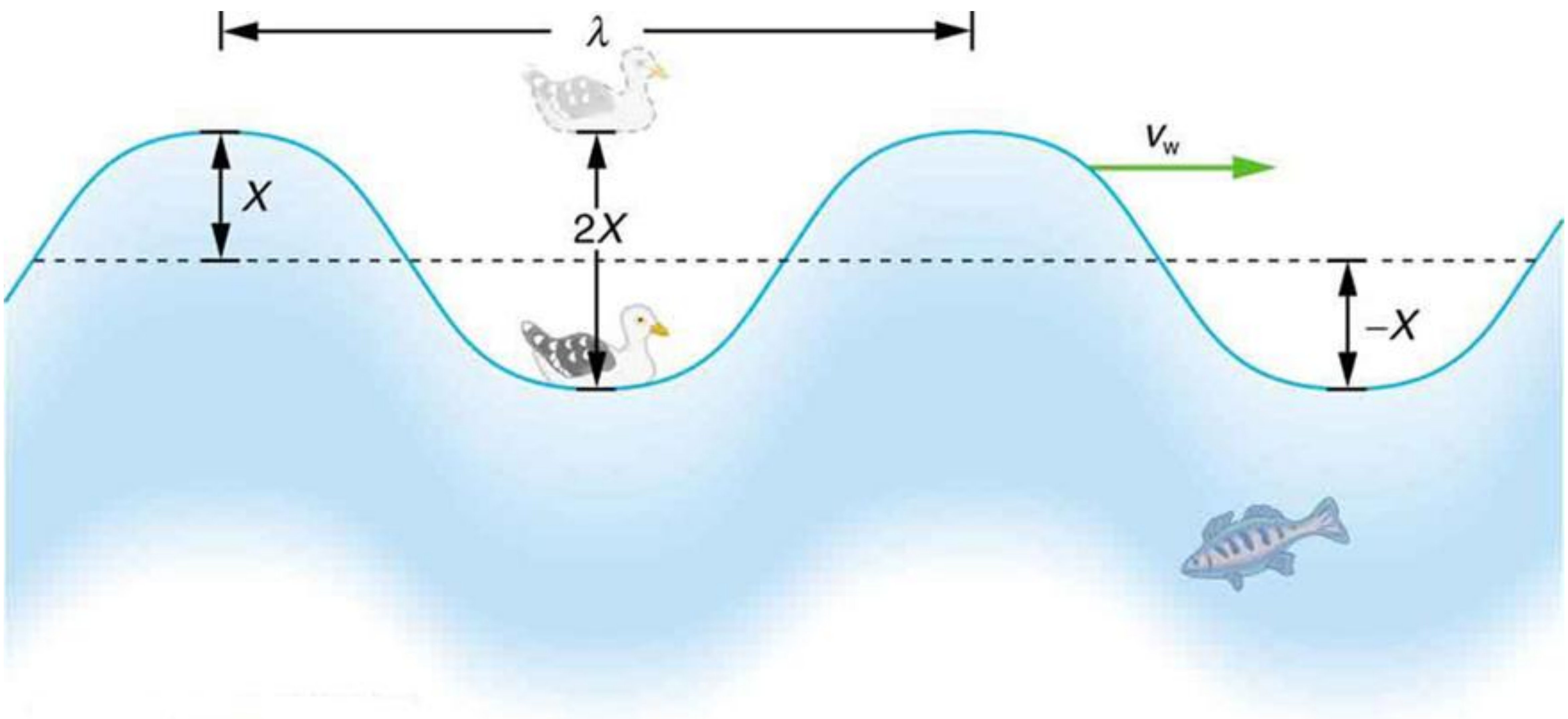


Linear harmonic oscillator

We just studied transverse waves. There are also longitudinal waves.

II. All of what we just did was at a fixed location along our wave, and allowing time to vary.

What if we did the opposite? That is, consider a fixed time and describe the wave at different locations. To describe this spatial variation we use $\psi(x) = A\cos(kx)$, here $k = 2\pi/\lambda$ is the "wave number" of the wave.



III. Really what we want to do is to model both the spatial and the time variations of a wave. What tool from advanced calculus do we need to do this? We'll use partial derivatives in particular.

The answer for this model is called the wave equation (1D):

$$\frac{\partial^2 \psi(x,t)}{\partial t^2} = c^2 \frac{\partial^2 \psi(x,t)}{\partial x^2}.$$

Let's guess the answer: $\psi(x - ct)$, any function whatsoever works as long as it only depends on the single variable $x - ct$.

Plug it in and see:

$$\frac{\partial \psi(x-ct)}{\partial t} = \psi'(x-ct)(-c), \ \frac{\partial^2 \psi(x-ct)}{\partial t^2} = \psi''(x-ct)c^2$$

$$\frac{\partial \psi(x-ct)}{\partial x} = \psi'(x-ct), \ \frac{\partial^2 \psi(x-ct)}{\partial x^2} = \psi''(x-ct). \text{ It works!}$$