Today

I. Office Hour Juggle
II. Questions that everyone can hear about the exam?
III. Last Time
IV. Waves in General
V. Intensity

I. No Yanpei Deng this week due to exam (will help in the lab. She's available MTuW from 7-8pm in Brody lab).

Not this week: Antu Antu will be providing homework support. Hours are: Tu 8-9pm, Th10:30-11:30am, Th 8-9pm.

First exam is a take-home exam, it was posted last Friday and is due the following Thursday at 11:59pm.

Thursday office hours will now run 5-6:30pm.

I. Talked about different parts of waves that we were using:

We used simple harmonic motion (SHM):

$$\frac{d^2\psi}{dt^2} = -\frac{k}{m}\psi,$$

$$\psi(t) = A\cos(\omega t), \ \ \omega = \sqrt{k/m}.$$

We also studied the position dependence of the way and we realized that was also harmonic

$$\psi(x,t) = A\cos(\omega t - kx),$$

A = amplitude, k = wave number = $2\pi/\lambda$, $\lambda = wavelength$,

ω = angular frequency = $2\pi f$, f = frequency

$[\omega]$ = rad/s, $[f]$=Hertz=Hz.

Introduced and studied the wave equation:

$$\frac{\partial^2\psi}{\partial t^2} = c^2\frac{\partial^2\psi}{\partial x^2}$$

Here c is to be understood as the speed of the wave.

III. Introduced and studied the wave equation:

$$\frac{\partial^2 \psi}{\partial t^2} = c^2 \frac{\partial^2 \psi}{\partial x^2}$$

Here c is to be understood as the speed of the wave.

We noticed that any function (whose derivative can be compute twice) is a solution if and only if it is a function of the single variable

$$\psi = \psi(x - ct).$$

Last time we checked this by computing derivatives.

IV. Today let's check this for the solution

$$\psi(x, t) = Ae^{i(kx - \omega t)} = Ae^{ik(x - \frac{\omega}{k}t)}.$$

This is a solution to the wave equation if and only if

$$c = \omega/k = 2\pi f/(2\pi/\lambda) = \lambda f.$$

Another completely general solution is

$$\psi = \psi(x + ct).$$

III. Introduced and studied the wave equation:

$$\frac{\partial^2 \psi}{\partial t^2} = c^2 \frac{\partial^2 \psi}{\partial x^2}$$

Let's check that this

$$\psi = \psi(x + ct),$$

Is a solution. First we

$$\frac{\partial \psi}{\partial t} = \psi'(x + ct)c, \ \frac{\partial^2 \psi}{\partial t^2} = \psi''(x + ct)c^2$$

$$\frac{\partial \psi}{\partial x} = \psi'(x + ct), \ \frac{\partial^2 \psi}{\partial x^2} = \psi''(x + ct) \cdot 1$$

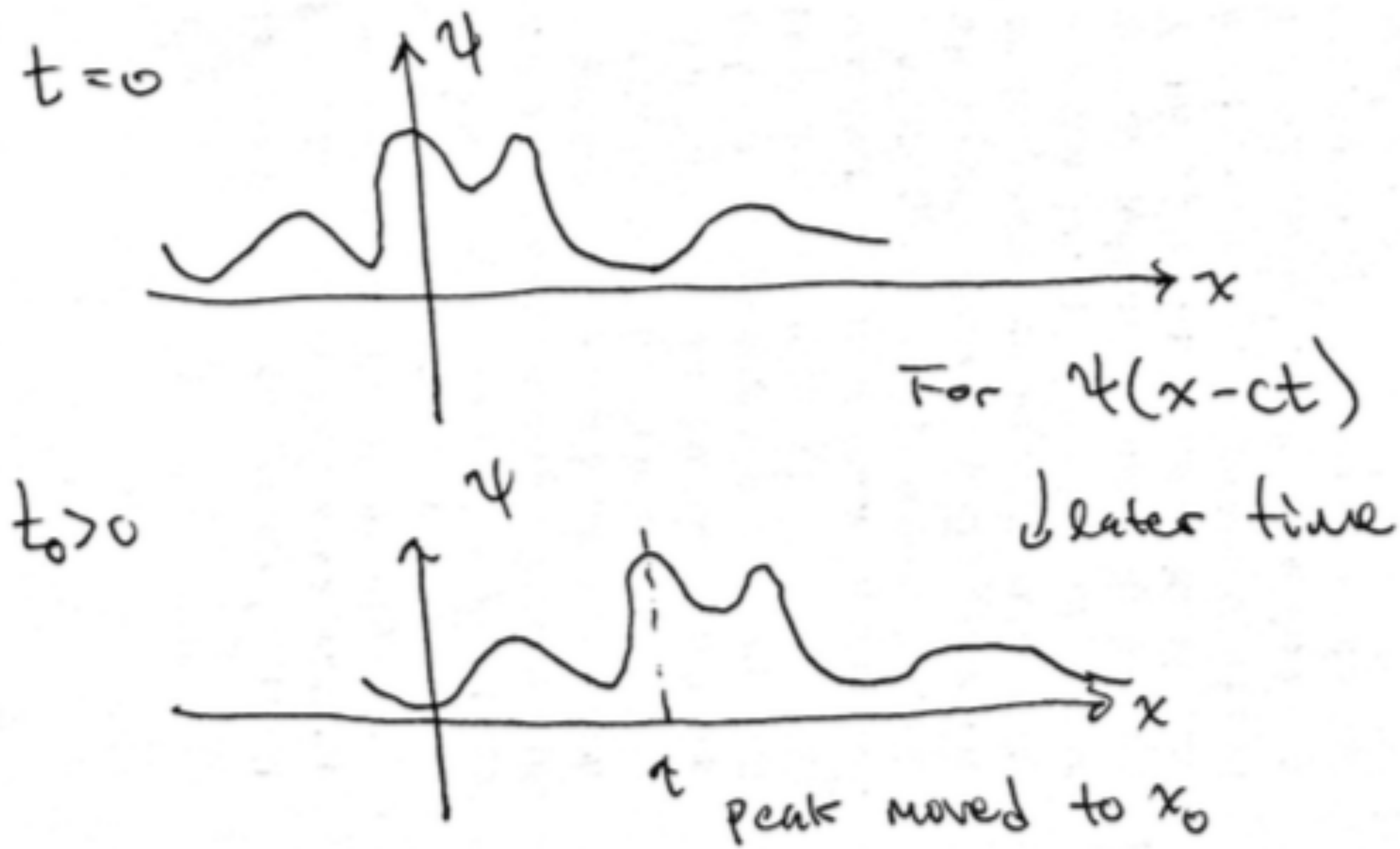
Plugging these into the wave equation we see that this is indeed a solution to the wave equation!

$$\psi(x, t) = \sqrt{k(x - ct)},$$

$$\psi(x, t) = a + bk(x + ct) + ck^2(x + ct)^2 + dk^3(x + ct)^3$$

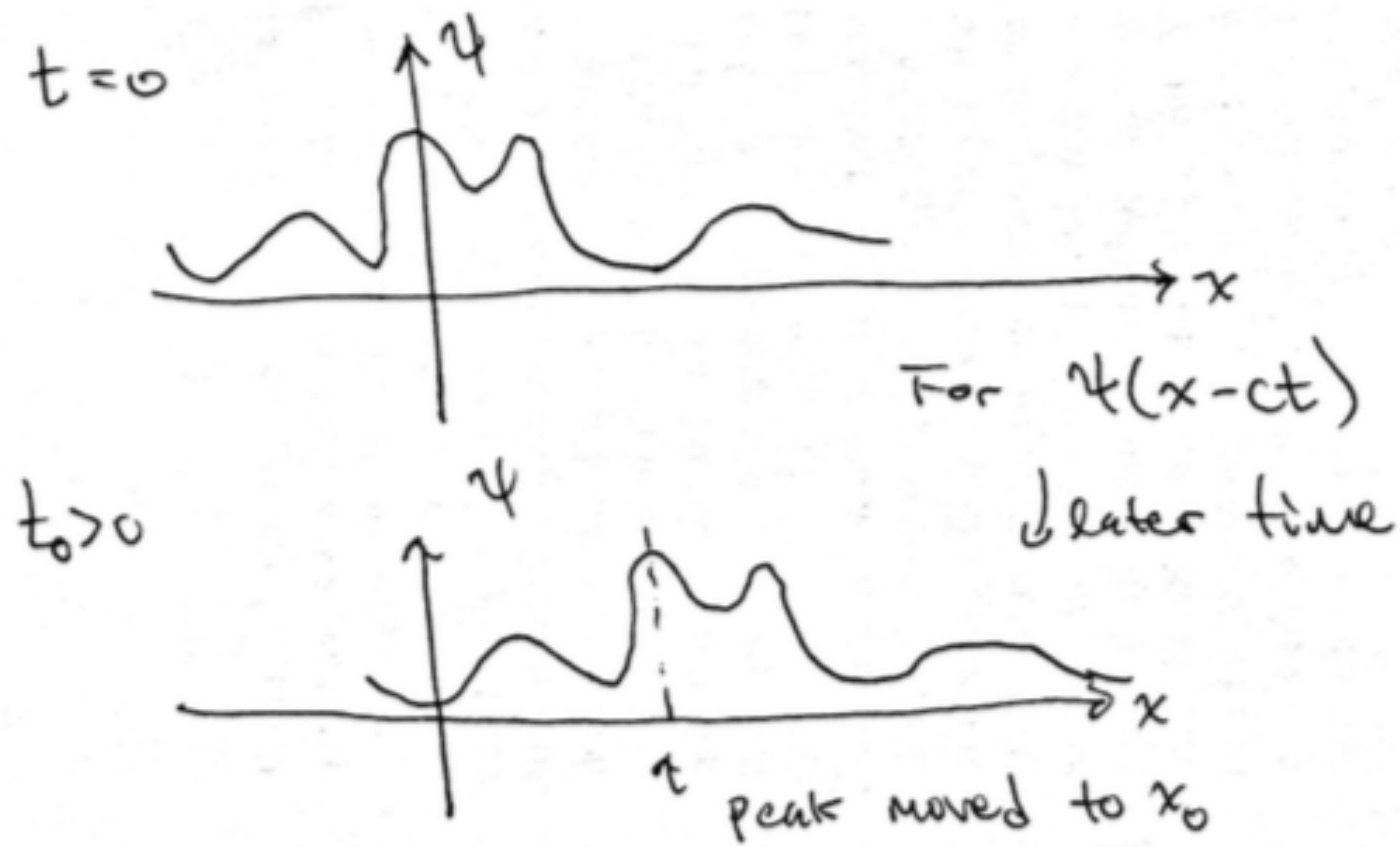
$$\psi(x, t) = Ae^{-k^2(x \pm ct)^2}$$

III. Let's draw some of these waves



As t advances the product $-ct$ gets more negative, so if I want to look at the same point on the wave I have to go to larger x. That means the wave is moving forward, towards $+x$, as time advances.

III. Let's draw some of these waves



$\psi(x + x_0, t_0) = \psi(x + x_0 - ct_0) = \psi(x,0) = \psi(x)$ then $x_0 - ct_0 = 0$ and $\dfrac{x_0}{t_0} = c$. In a very similar manner you can prove that the wave

$\psi(x, t) = \psi(x + ct)$ is moving to the left.

III. Solutions of the wave equation can be added to get another solution! We call this very important property "superposition of waves": suppose ψ_1 is a solution, and $\psi_2 isasolution$

And consider $\psi \equiv \psi_1 + \psi_2$,

$$\frac{\partial^2 \psi}{\partial t^2} = c^2 \frac{\partial^2 \psi}{\partial x^2}$$

Then we have

$$\frac{\partial^2 \psi_1}{\partial t^2} + \frac{\partial^2 \psi_2}{\partial t^2} = c^2 \left(\frac{\partial^2 \psi_1}{\partial x^2} + \frac{\partial^2 \psi_2}{\partial x^2} \right). \checkmark$$

A (useful) subset of all solutions are the "periodic waves:

$\psi(x + \lambda, t) = \psi(x, t)$ wavelength periodicity

Surprisingly, if a wave is periodic in space it is also periodic in time:

$f\lambda = c$ so that $f = \dfrac{c}{\lambda}$, this leads us to the period $\tau = \dfrac{1}{f}$.

IV.

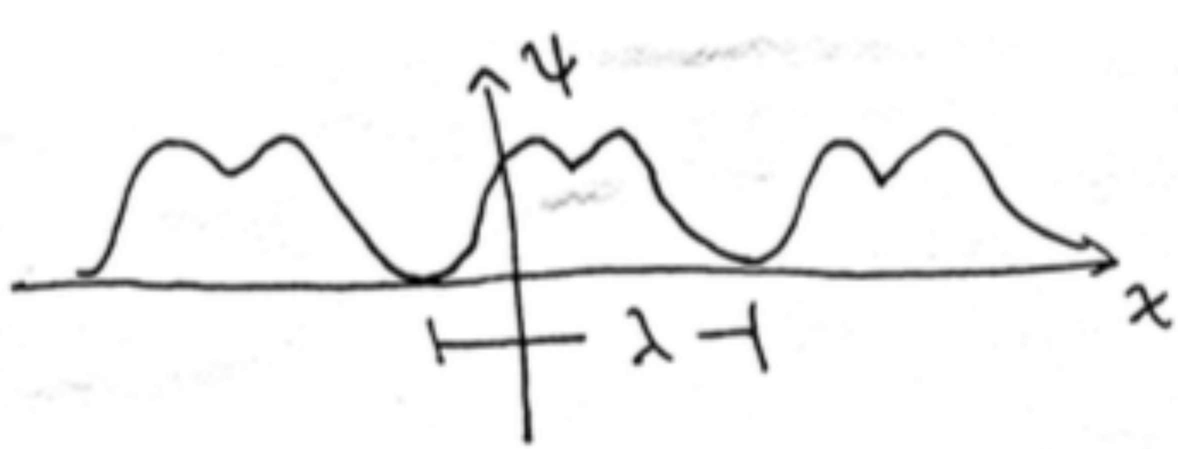
A (useful) subset of all solutions are the "periodic waves:

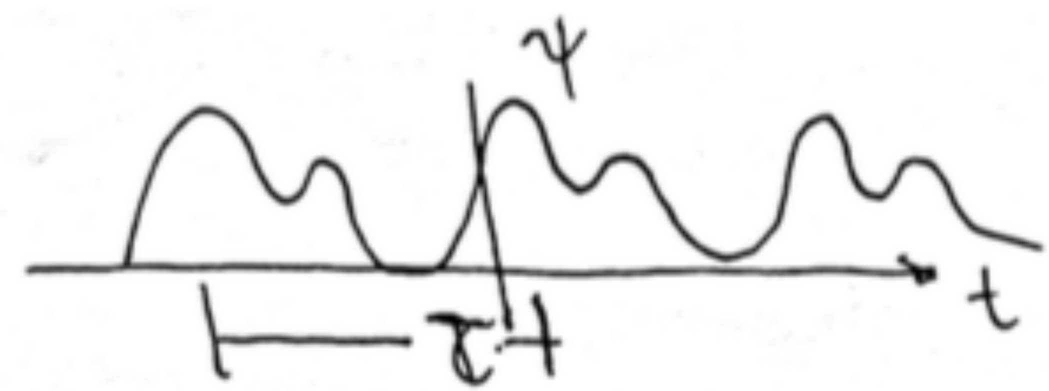
$\psi(x + \lambda, t) = \psi(x, t)$ wavelength periodicity

Surprisingly, if a wave is periodic in space it is also periodic in time:

$f\lambda = c$ so that $f = \dfrac{c}{\lambda}$, this leads us to the period $\tau = \dfrac{1}{f}$.

We can picture these this way:





IV. An even more special subset are the "harmonic waves" Harmonic waves are the sinusoidal solutions of the wave equation. Of course, these are just "phase" offsets of one another

$$\psi(x,t) = A\cos(k[x \pm ct] + \phi)$$

A=amplitude, k=wave number, ϕ=phase or phase shift.

I. Last Time

Explored complex numbers:

$z^* = (x + iy)^* = x - iy$

$Re(z) = \frac{1}{2}(z + z^*), \quad Im(z) = \frac{1}{2i}(z - z^*)$

$z = re^{i\theta}$, and Euler's identity $\boxed{e^{i\theta} = \cos\theta + i\sin\theta.}$

II. Einstein's 2nd postulate highlights the special nature of light. But, so far we've treated light as a particle. Part of the discovery of light's character came through its wave properties, to which we now turn.

The waves most familiar from our surroundings are the motion of energy and momentum through a medium; e.g. sound, water, in wood (marimba), through rock (earthquake!)

II.

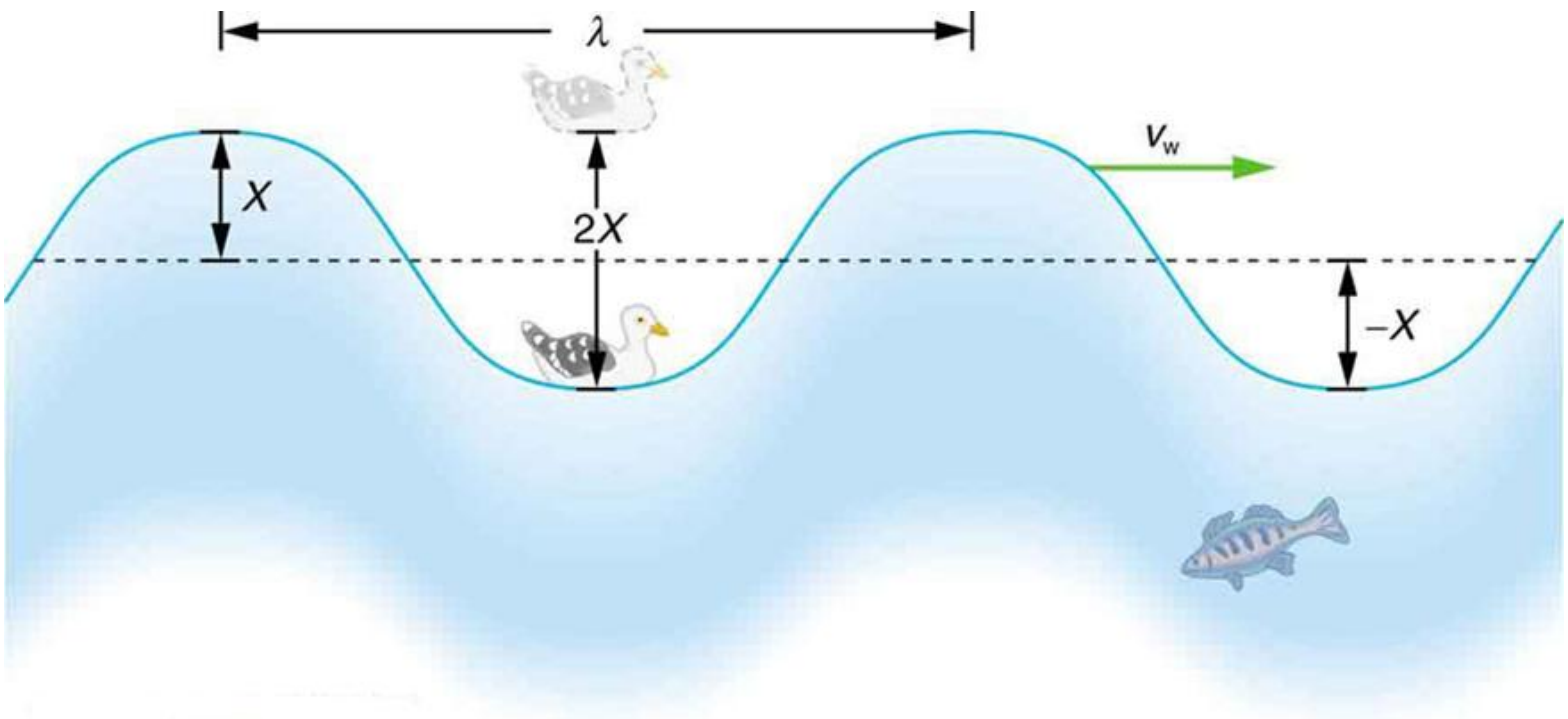
The waves most familiar from our surroundings are the motion of energy and momentum through a medium; e.g. sound, water, in wood (marimba), through rock (earthquake!).

If light is an electromagnetic wave, what is the medium it is waving? This is the question that Michelson and Morley's experiment hinted at answering. (People at the time called this the luminiferous aether.)

II.

How will the duck move? The duck oscillates up and down, up and down. How do we model this motion?

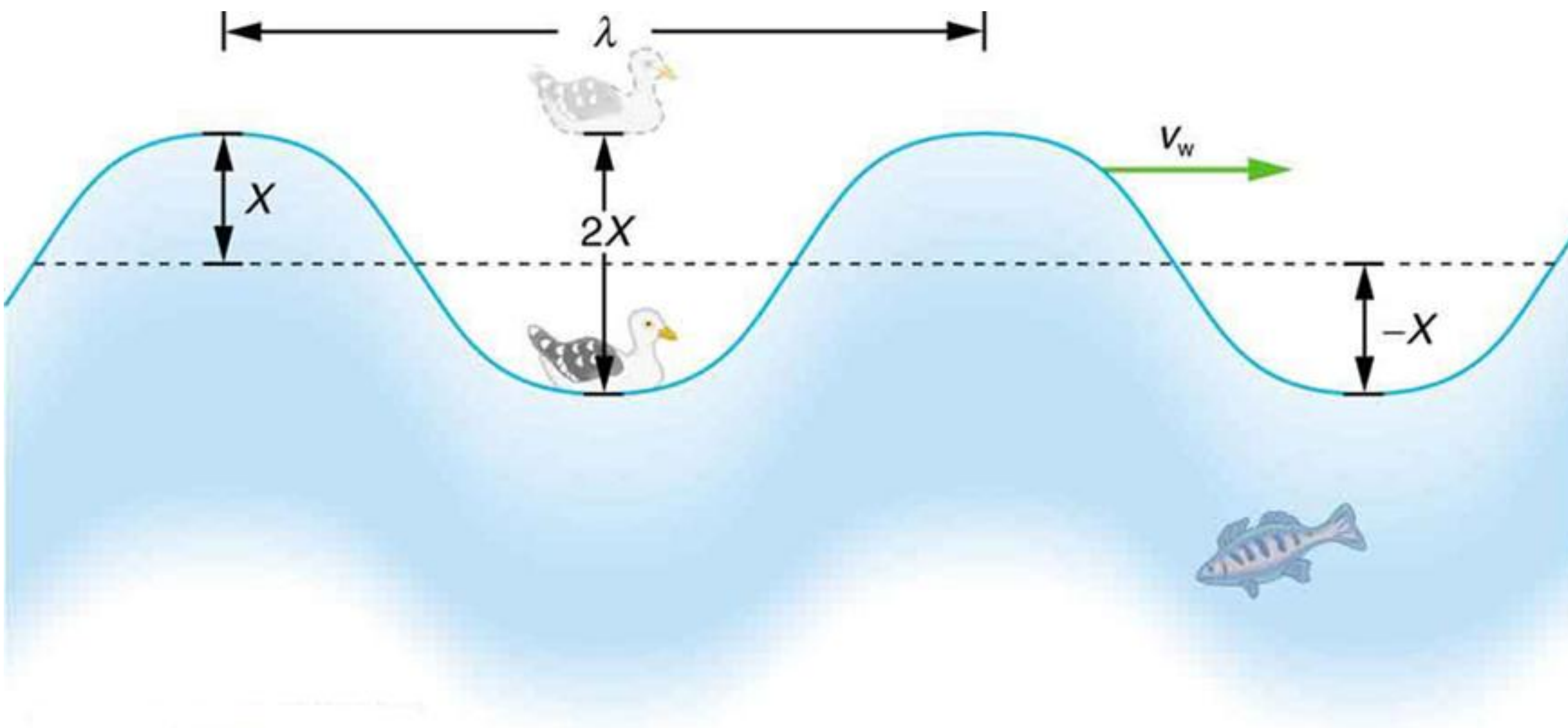
$\psi(t) = A\cos(\omega t + \phi)$, here ω is the "angular frequency" $\omega = 2\pi f$, where f is the standard "frequency" $[f]$=Hertz=1/seconds.



II. According to Newton $F = ma$ and in this case the force is given by a linear restoring force, "Hooke's Law", $F = -k\psi$. Then we have:

$$ma = m\frac{d^2\psi}{dt^2} = -k\psi(t),$$

To get the answer from the previous slide I have to solve a differential equation!

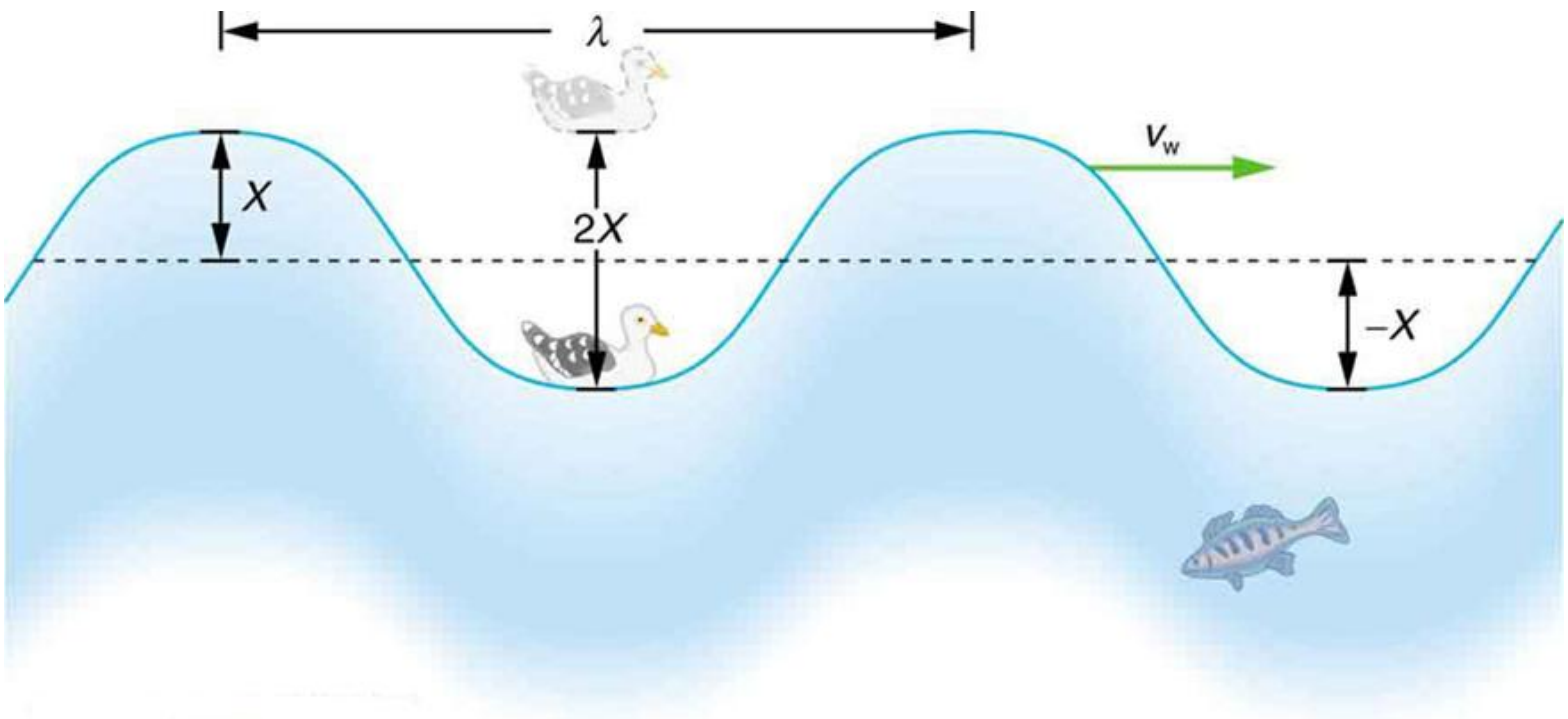


II. Equation of motion of the duck: $ma = m\dfrac{d^2\psi}{dt^2} = -\,k\psi(t)$,

Let's guess $\psi(t) = e^{i\omega t} = \cos(\omega t) + i\sin(\omega t)$, then

$\dfrac{d\psi}{dt} = i\omega e^{i\omega t}$, $\dfrac{d^2\psi}{dt^2} = -\,\omega^2 e^{i\omega t} = -\,\omega^2\psi(t)$ is a solution if and only if

$\omega^2 = k/m$ or, in other words, $\omega = \sqrt{k/m}$.

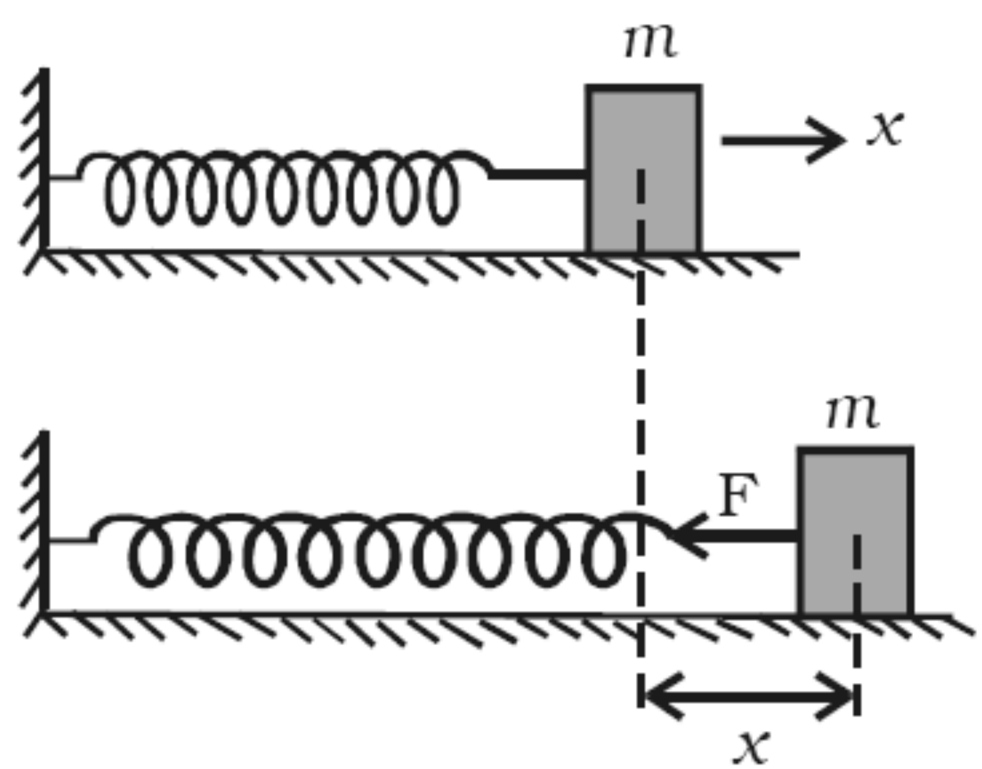


II. Equation of motion of the duck: $ma = m\dfrac{d^2\psi}{dt^2} = -\,k\psi(t)$,

Let's guess $\psi(t) = e^{i\omega t} = \cos(\omega t) + i\sin(\omega t)$, then

$\dfrac{d\psi}{dt} = i\omega e^{i\omega t}$, $\dfrac{d^2\psi}{dt^2} = -\,\omega^2 e^{i\omega t} = -\,\omega^2\psi(t)$ is a solution if and only if

$\omega^2 = k/m$ or, in other words, $\omega = \sqrt{k/m}$.

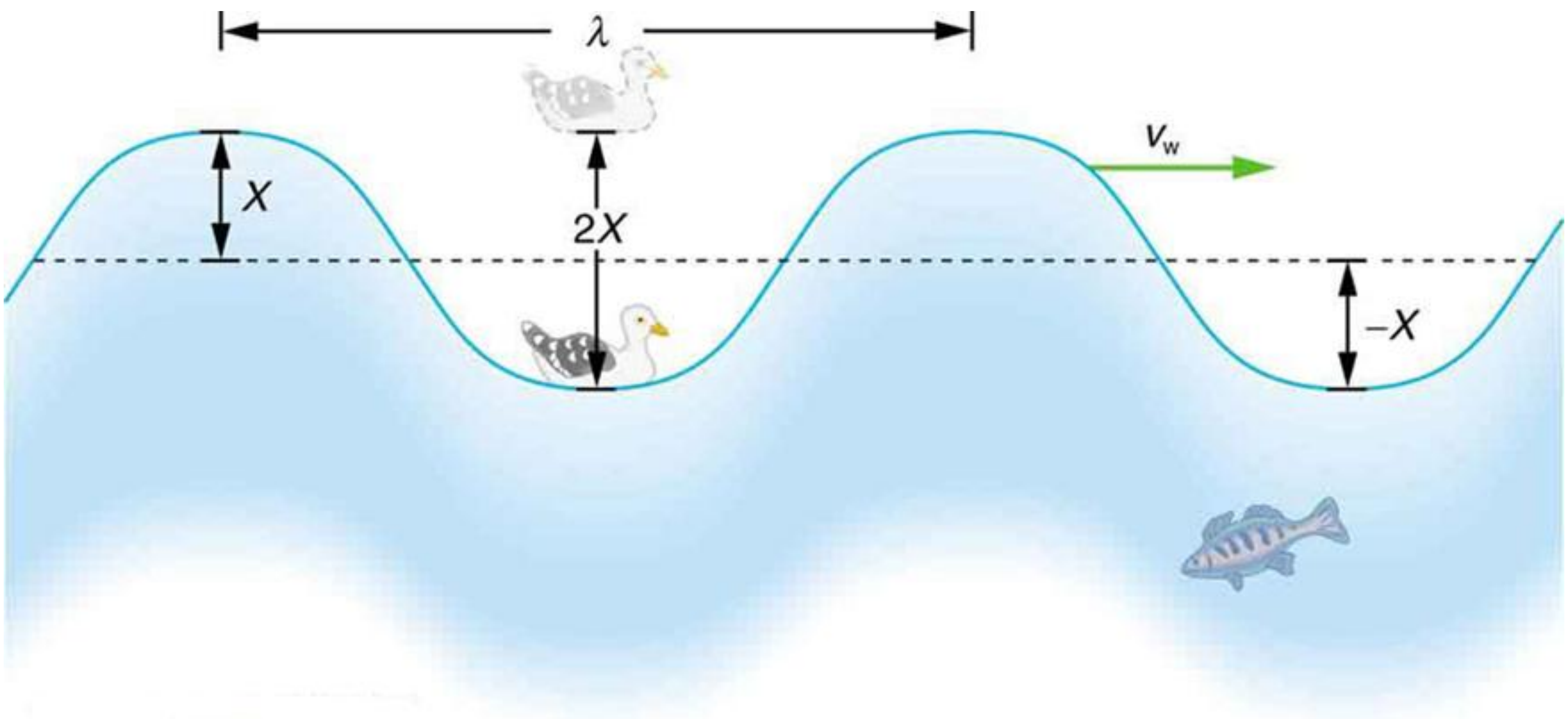


Linear harmonic oscillator

We just studied transverse waves. There are also longitudinal waves.

II. All of what we just did was at a fixed location along our wave, and allowing time to vary.

What if we did the opposite? That is, consider a fixed time and describe the wave at different locations. To describe this spatial variation we use $\psi(x) = A\cos(kx)$, here $k = 2\pi/\lambda$ is the "wave number" of the wave.



III. Really what we want to do is to model both the spatial and the time variations of a wave. What tool from advanced calculus do we need to do this? We'll use partial derivatives in particular.

The answer for this model is called the wave equation (1D):

$$\frac{\partial^2 \psi(x,t)}{\partial t^2} = c^2 \frac{\partial^2 \psi(x,t)}{\partial x^2}.$$

Let's guess the answer: $\psi(x - ct)$, any function whatsoever works as long as it only depends on the single variable $x - ct$.

Plug it in and see:

$$\frac{\partial \psi(x-ct)}{\partial t} = \psi'(x-ct)(-c),\ \frac{\partial^2 \psi(x-ct)}{\partial t^2} = \psi''(x-ct)c^2$$

$$\frac{\partial \psi(x-ct)}{\partial x} = \psi'(x-ct),\ \frac{\partial^2 \psi(x-ct)}{\partial x^2} = \psi''(x-ct).$$ It works!