<u>Today</u>

I. Last Time
II. Waves and Intensity
III. Intensity and Complex Numbers

I. Yanpei Deng this week due to exam (will help in the lab. She's available MTuW from 7-8pm in Brody lab).

This week: Antu Antu will be providing homework support. Hours are: Tu 8-9pm, Th10:30-11:30am, Th 8-9pm.

Thursday office hours will now run 5-6:30pm.

I. The wave equation and its solutions.

Several special classes of solutions:

1. Periodic solutions: $\psi(x+\lambda, t+\tau) = \psi(x,t)$

The parameters λ and τ are related through the wave speed

$$c = \frac{\lambda}{\tau} = \lambda f = \lambda \frac{\omega}{2\pi} = \frac{2\pi}{k}\frac{\omega}{2\pi} = \frac{\omega}{k}.$$

2. Harmonic solutions: $\psi(x,t) = A\cos(k[x \pm ct] + \phi)$,

$A =$ amplitude, ϕ=phase shift.

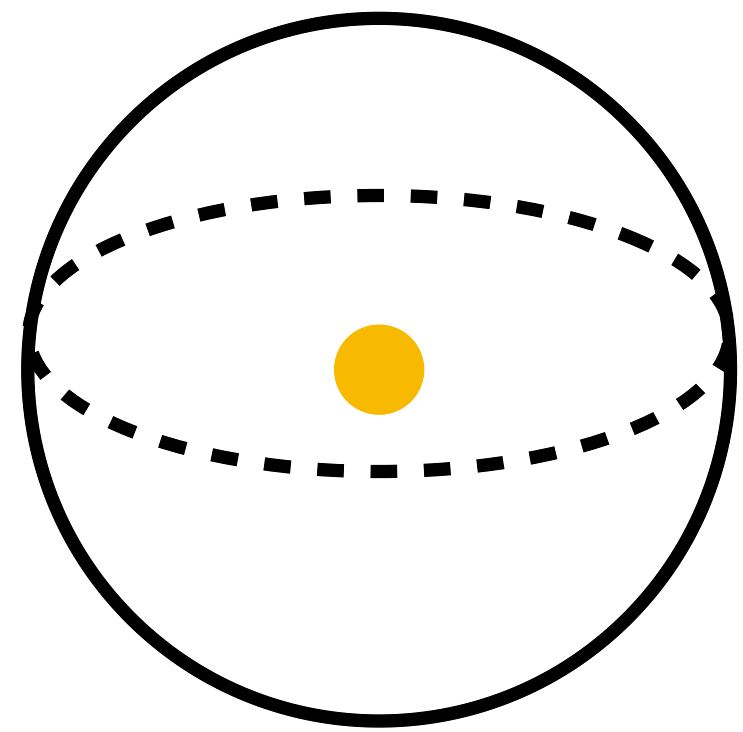
Notice that the harmonic solutions are periodic! Harmonic waves are useful:

1. sin/cos are easy to manipulate (even better $e^{i\theta}$)
2. Any periodic wave = sum (possibly infinite) of harmonic waves (Fourier analysis).
3. Any wave over a finite range of x & t can be written as the sum of harmonic waves.

II. Intensity

Imagine the brightness of a star; it depends on the star's power = $\frac{\text{Energy}}{\text{time}}$, but also on how far away you are. This motivates us to define the "intensity"

$$\text{Intensity} = \frac{\text{Energy}}{\text{Time} \cdot \text{Area}} = \frac{\text{Power}}{\text{Area}} \equiv I.$$

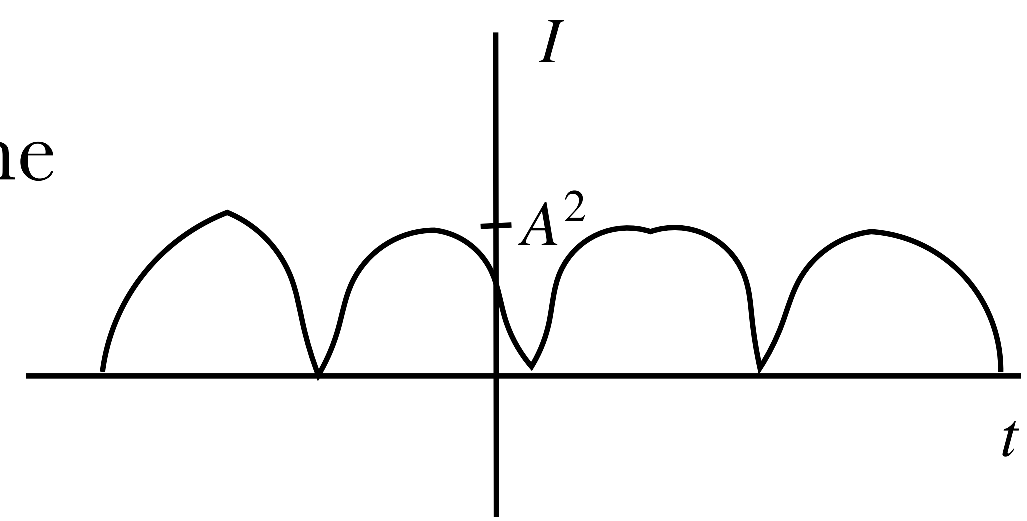


On the homework, you will show that the energy of a wave is proportional to its 2nd power and so its intensity is

$$I \propto \left(\psi(x,t)\right)^2 = A^2 \cos^2(kx - \omega t + \phi),$$

Intensity will generally have twice the frequency of the initial wave.

II. Intensity



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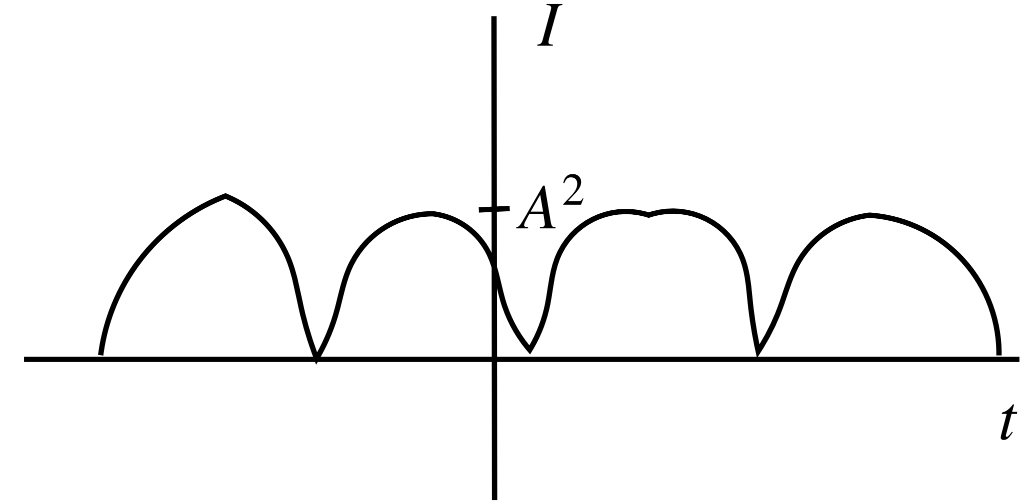
What's the frequency of an EM wave?

$$f = \frac{c}{\lambda} = \frac{3 \times 10^8 m/s}{300 \times 10^{-9} m} \sim 10^{15}\text{Hz}$$

Intensity measurements are almost always averages over many periods of the incoming signal:

$$I \propto \frac{1}{\tau}\int_0^\tau \psi(x,t)^2 dt$$

II. Intensity



Take a wave of the form

$$I \propto \left(\psi(x,t)\right)^2 = A^2\cos^2(kx - \omega t + \phi),$$

The average intensity is

$$I \propto \frac{1}{\tau}\int_0^\tau \psi(x,t)^2 dt.$$

Recall

$$e^{i\theta} = \cos\theta + i\sin\theta, \quad \text{then. } \cos\theta = Re(e^{i\theta}) = \frac{1}{2}(e^{i\theta} + e^{-i\theta}).$$

More generally if $\psi = Ae^{i(kx-\omega t+\phi)}$, then I want $\frac{1}{2}(\psi + \psi^*)$. Then we have

$$I \propto \frac{1}{\tau}\frac{1}{4}\int_0^\tau (\psi + \psi^*)^2 dt = \frac{1}{\tau}\frac{1}{4}\int_0^\tau (\psi^2 + \psi^{*2} + 2|\psi|^2)dt$$

II. Intensity

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$$I \propto \frac{1}{\tau}\frac{1}{4}\int_0^\tau A^2(e^{2i(kx-\omega t+\phi)} + e^{-2i(kx-\omega t+\phi)} + 2)dt$$

Pause for a second to compute

$$\int e^{ax+b}dx = \frac{1}{a}e^{ax+b}.$$

Then,

$$I \propto \frac{1}{\tau}\frac{A^2}{4}\left(\frac{1}{-2i\omega}e^{2i(kx-\omega t+\phi)} + \frac{1}{2i\omega}e^{-2i(kx-\omega t+\phi)} + 2t\right)_0^\tau$$

II. Intensity

Then,

$$I \propto \frac{1}{\tau}\frac{A^2}{4}\left(\frac{1}{-2i\omega}e^{2i(kx-\omega t+\phi)} + \frac{1}{2i\omega}e^{-2i(kx-\omega t+\phi)} + 2t\right)_0^\tau$$

$$I \propto \frac{1}{\tau}\frac{A^2}{4}\left(\frac{1}{-2i\omega}e^{2i(kx-\omega\tau+\phi)} + \frac{1}{2i\omega}e^{-2i(kx-\omega\tau+\phi)} + 2\tau\right) - \frac{1}{\tau}\frac{A^2}{4}\left(\frac{1}{-2i\omega}e^{2i(kx-\omega 0+\phi)} + \frac{1}{2i\omega}e^{-2i(kx-\omega 0+\phi)} + 2(0)\right)$$

Cancel **Cancel**

We get the incredibly simple result:

$$I \propto \frac{A^2}{2}.$$

Another way of seeing the cancellation is $\omega = \frac{2\pi}{\tau}$, then $\omega\tau = 2\pi$

and $e^{i2\pi} = cos(2\pi) + i\sin(2\pi) = 1.$