Today

I. Last Time
II. Attenuation
III. Interference of waves with a phase difference
IV. Complex index of refraction

I. Yanpei Deng this week due to exam (will help in the lab. She's available MTuW from 7-8pm in Brody lab).

This week: Antu Antu will be providing homework support. Hours are: Tu 8-9pm, Th10:30-11:30am, Th 8-9pm.

Thursday office hours will now run 5-6:30pm.

I. Last time we discussed Intensity, qualitatively this is the "strength" or "brightness" of a signal. But, quantitatively it is defined by the power per unit area. What area we are talking about depends on the context of the measurement.

Most of the time when we talk about intensity we mean the average intensity. (From now on if I say intensity without a modified, I will always mean average intensity.)

We also took an explicit harmonic wave and computed its intensity

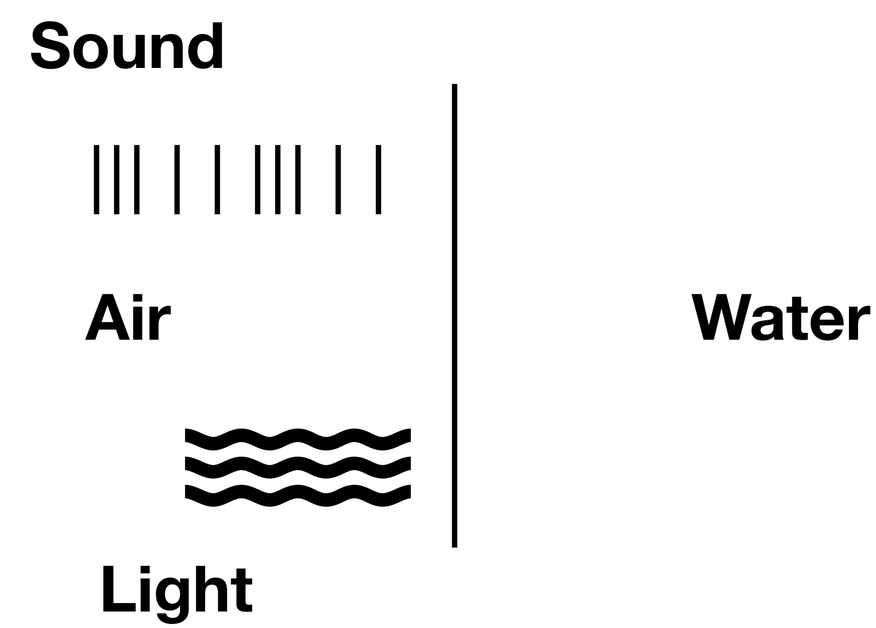
$$I \propto \frac{1}{\tau}\int_0^{\tau} \psi^2(x,t)dt \propto \frac{1}{2}A^2.$$

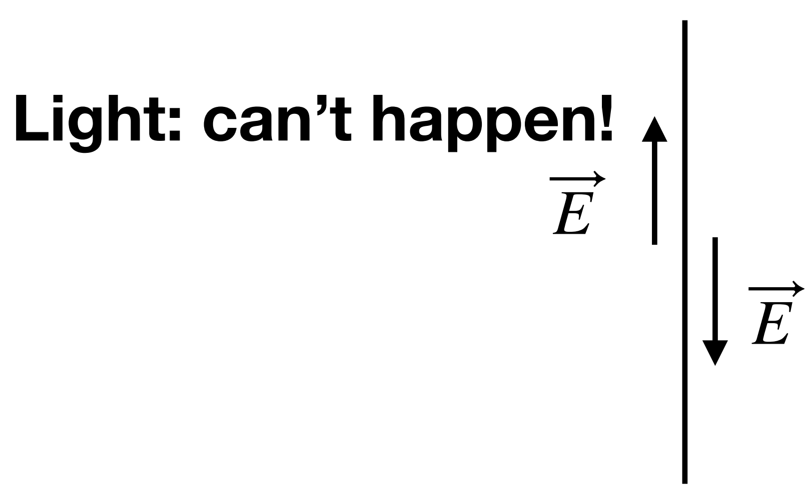
Recall that we were leveraging the complex notation:

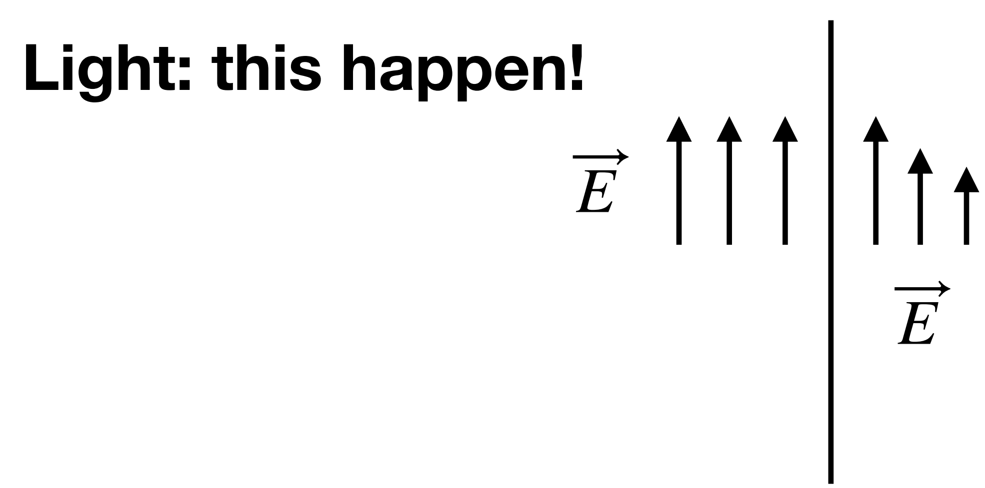
$$\psi(x,t) = A\cos(kx - \omega t + \phi) \rightarrow \tilde{\psi}(x,t) = Ae^{i(kx-\omega t+\phi)}.$$

II. Attenuation: What is the most important parameter in a harmonic wave? We'll focus for a little while on ω. It's an important parameter (more so than λ) because ω doesn't change when we go from one medium to another:

Consider the case of light and concentrate on its electric field piece. Right at the boundary between The materials the field should be continuous.





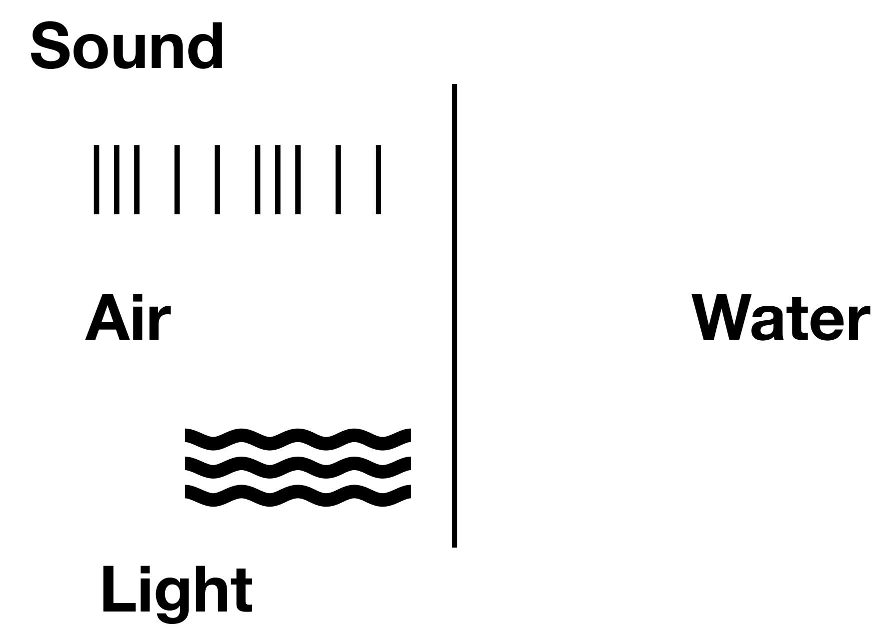


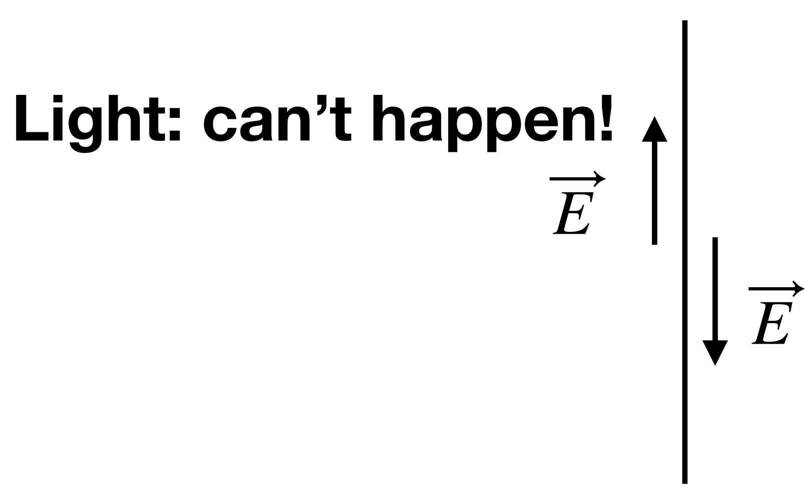
II. Attenuation: We need continuity at boundary for whatever quantity the wave amplitude is describing. This amplitude can change as you move deeper into the material, but at the boundary it should be continuous.

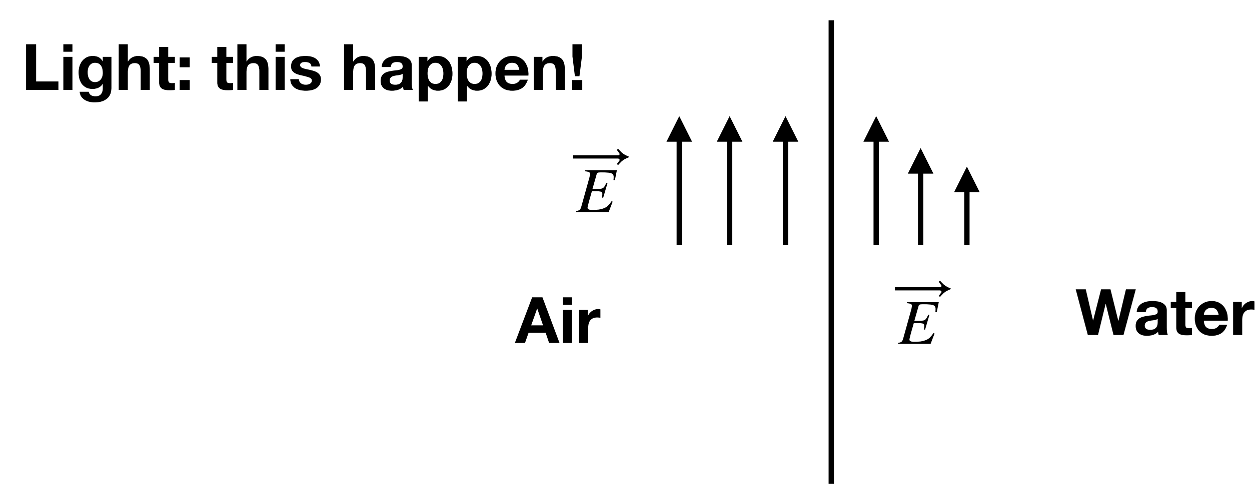
The speed can change at a boundary:

$$v = \frac{c}{n},$$

here n is the “index of refraction”.







II. Attenuation is the effect of a wave getting smaller and smaller in amplitude as it enters a material. Let's take the reasonable guess that for each meter of material that the wave travels into, it loses the same amount of amplitude.

For example, if the amplitude decreases by 10% over 1m, what is it at 3m?

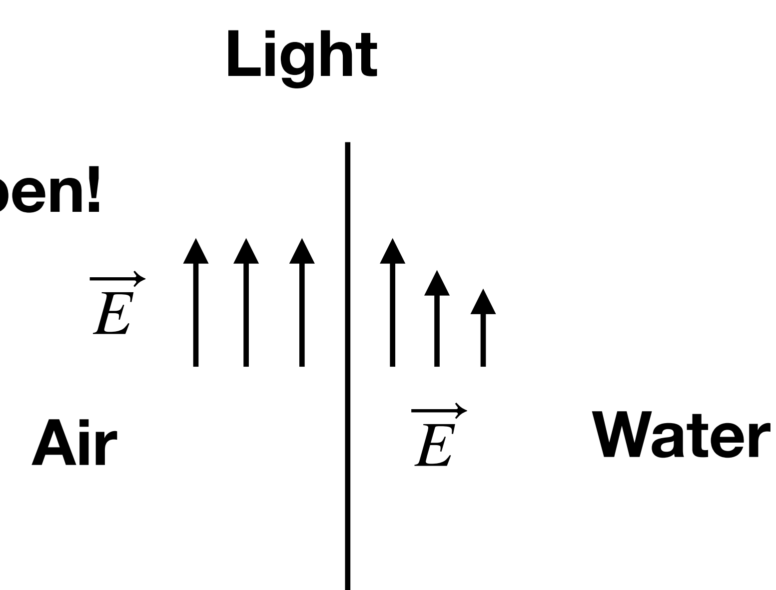
Say, the initial amplitude is A_0, then at 1m

$A(1m) = A_0 - 0.1A_0 = 0.9A_0$

$A(2m) = (1 - 0.1)^2 A_0$

$A(3m) = (1 - 0.1)^3 A_0$.

Let's consider a continuous version of this question. Consider the percentage lost over distance dx some constant with units length^{-1}, call this α.



II. Let's consider a continuous version of this question. Consider the percentage lost over distance dx some constant with units length^{-1}, call this α. Then

$$A(x+dx) = A(x)(1-\alpha dx) = A(x) - A(x)\alpha dx.$$

This gives

$$\frac{A(x+dx)-A(x)}{dx} = -\alpha A(x),$$

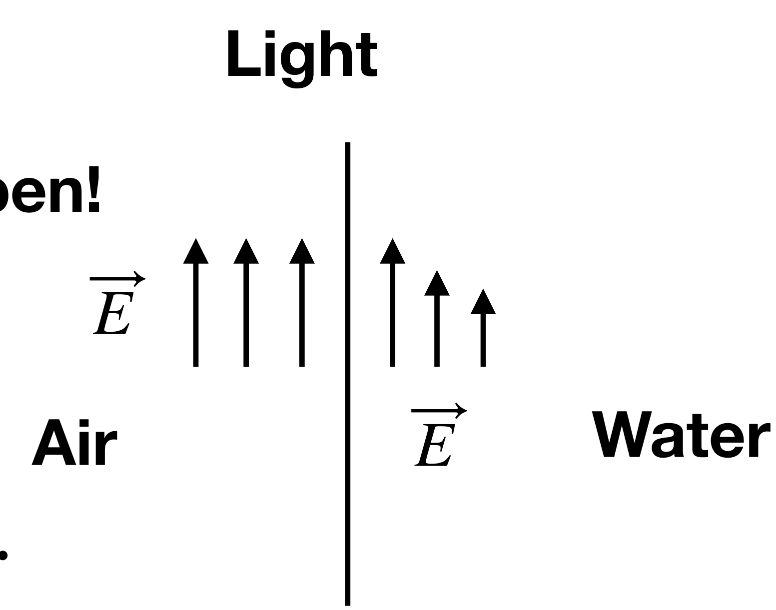
here the left hand side is dA/dx and we have

$$\frac{dA}{dx} = -\alpha A(x).$$

We can solve this by

`separation of variables'

Light: this happen!



$$\frac{dA}{A} = -\alpha dx,$$

$$\int \frac{dA}{A} = \int -\alpha dx = -\alpha \int dx = -\alpha x + \text{const.}$$

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We can also do the integral on the left to get

$\ln A = -\alpha x + \text{const.}$ Exponentiating gives

$A(x) = e^{-\alpha x + \text{const.}} = \text{const.} e^{-\alpha x}$.

We can fix the arbitrary constant out front

by using the initial condition

$A(x) = A(0)e^{-\alpha x}$.

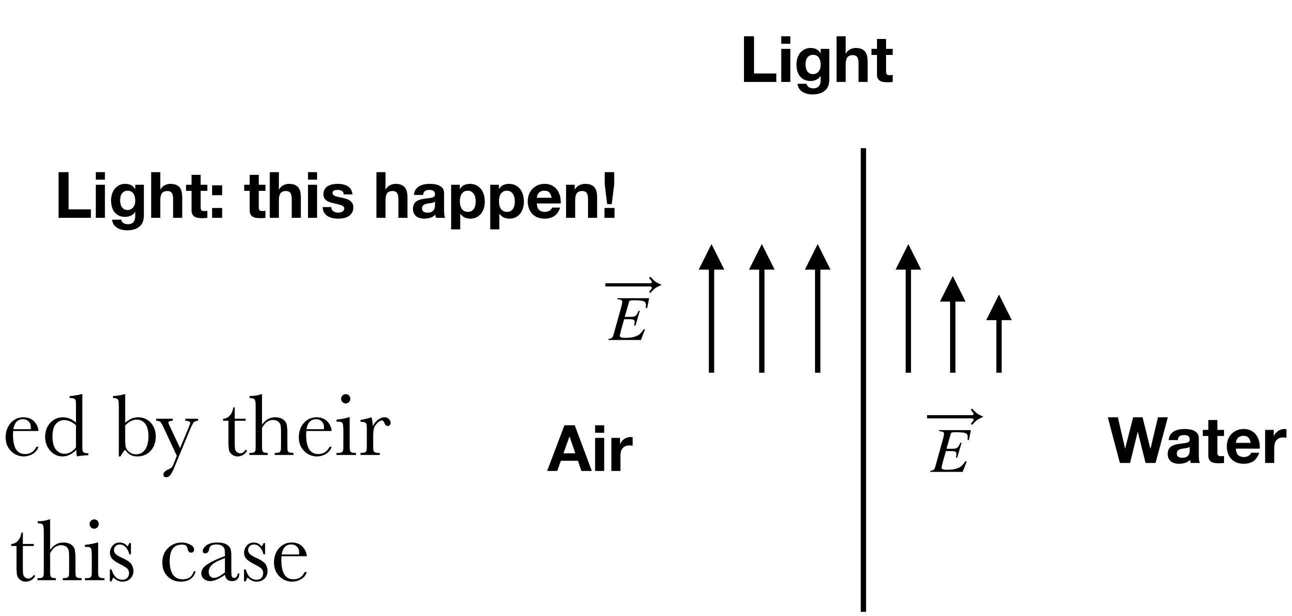
All the time in physics

we see exponential decays.

These are often characterized by their

`characteristic values' or in this case

the skin depth of the material.



II. These are often characterized by their `characteristic values' or in this case the skin depth of the material. This is defined as how far you have to go into the material for the amplitude to fall by a factor of $1/e \approx 1/3$, then we have that

$$\text{skin depth} = \frac{1}{\alpha}.$$

III. What is the intensity of a superposition of harmonic waves with same E_0, k, ω, but different ϕ?

First, the wave is as follows

$E(x,t) = E_0 \cos(kx - \omega t) + E_0 \cos(kx - \omega t + \phi)$.

The intensity is

$I = \epsilon_0 c \overline{E^2} = \epsilon_0 c E_0^2 \overline{[\cos(kx - \omega t) + \cos(kx - \omega t + \phi)]^2}$

$I = \epsilon_0 c E_0^2 \overline{[\cos^2(kx - \omega t) + \cos^2(kx - \omega t + \phi) + 2\cos(kx - \omega t)\cos(kx - \omega t + \phi)]}$

$I = \epsilon_0 c E_0^2 [\frac{1}{2} + \frac{1}{2} + 2\overline{\cos(kx - \omega t)\cos(kx - \omega t + \phi)}]$