Today

I. Last Time
II. Interference of waves with a phase difference
III. Complex index of refraction
IV. Diffusion Equation

I. Yanpei Deng this week due to exam (will help in the lab. She's available MTuW from 7-8pm in Brody lab).

This week: Antu Antu will be providing homework support. Hours are: Tu 8-9pm, Th10:30-11:30am, Th 8-9pm.

Today's office hours 5:30-7pm.
Thursday office hours will now run 5-6:30pm.

I. Last time we started to discuss the intensity of a superposition of harmonic waves. We ended right in the midst of that calculation and part of today will be devoted to completing it.

Discussed attenuation in a lot of detail. Made a connection with our first M&M lab where we had a fixed number of decays for each trial. In the case of waves we lose a fixed percentage of the amplitude for every unit of distance that we travel into the medium:

$$A(x) = A(0)e^{-\alpha x}, \text{ (attenuation equation).}$$

Of course, it is only approximately true in some specific materials that we lose the same amplitude at every step. We describe this attenuation with the "characteristic length" or "skin depth", which is $\ell = 1/\alpha$.

We noticed that it is ω that it constant across a boundary, not λ.

II. What is the intensity of a superposition of harmonic waves with same E_0, k, ω, but different ϕ?

First, the wave is as follows

$E(x,t) = E_0 \cos(kx - \omega t) + E_0 \cos(kx - \omega t + \phi)$.

The intensity is

$$I = \epsilon_0 c \overline{E^2} = \epsilon_0 c E_0^2 \overline{[\cos(kx - \omega t) + \cos(kx - \omega t + \phi)]^2}$$

$$I = \epsilon_0 c E_0^2 \overline{[\cos^2(kx - \omega t) + \cos^2(kx - \omega t + \phi) + 2\cos(kx - \omega t)\cos(kx - \omega t + \phi)]}$$

$$I = \epsilon_0 c E_0^2 [\frac{1}{2} + \frac{1}{2} + 2\overline{\cos(kx - \omega t)\cos(kx - \omega t + \phi)}]$$

Going from here we have to figure out what to do with this last term: recall $\cos A \cos B = \frac{1}{2}\cos(A+B) + \frac{1}{2}\cos(A-B)$. Then

$$I = \epsilon_0 c E_0^2 [\frac{1}{2} + \frac{1}{2} + 2\overline{\left[\frac{1}{2}\cos(-\phi) + \frac{1}{2}\cos(2kx - 2\omega t + \phi)\right]}]$$

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For $\phi = 0$ we get twice the amplitude and four times the intensity!

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For $\phi = 0$ we get twice the amplitude and four times the intensity! The case $\phi = \pi$ gives completely destructive interference.

III. Let's return to our wave that experiences attenuation, $A(x)$. Let's also assume that it's a harmonic wave

$$\psi(x,t) = A(x)\cos(kx - \omega t) = A_0 e^{-\alpha x}\cos(kx - \omega t)$$

$$\rightsquigarrow \tilde{\psi}(x,t) = A_0 e^{-\alpha x} e^{i(kx-\omega t)} = A_0 e^{-\alpha x + i(kx - \omega t)} = A_0 e^{i(i\alpha x + kx - \omega t)} = A_0 e^{i([k+i\alpha]x - \omega t)}$$

This is just like the harmonic waves we started with if we let $\tilde{k} = k + i\alpha$. Then we have a super compact wave: $\tilde{\psi}(x,t) = A_0 e^{i(\tilde{k}x - \omega t)}$.

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Recall that the wave speed is given by:

$c = \lambda f = \dfrac{2\pi}{k}\dfrac{\omega}{2\pi} = \dfrac{\omega}{k}$. Empirically we measure that the speed of light decreases when you enter a medium by a fixed factor, the index of refraction n:

$v_{\text{medium}} = \dfrac{c}{n} = \dfrac{\omega}{k}$, so k must change when we enter the medium.

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Apparently we can introduce a complex index of refraction:
$\tilde{n} = \frac{\tilde{k}c}{\omega} = \frac{kc}{\omega} + i\frac{\alpha c}{\omega} = n + i\kappa$, note that $\kappa \equiv \frac{\alpha c}{\omega}$,
here κ characterizes the absorption of the wave.