Today

I. Questions about Complex Waves
II. Last Time
III. Physical Origin of the Index of Refraction
IV. Diffusion Equation

I. Yanpei Deng this week due to exam (will help in the lab. She's available MW from 7-8pm in Brody lab).

This week: Antu Antu will be providing homework support. Hours are: Tu 8-9pm, Th10:30-11:30am, Th 8-9pm.

We noticed that by combining the real and imaginary exponents of our waves we could get a complex index of refraction:

$$\tilde{n} = \frac{\tilde{k}c}{\omega} = \frac{kc}{\omega} + i\frac{\alpha c}{\omega} = n + i\kappa, \text{ note that } \kappa \equiv \frac{\alpha c}{\omega}.$$

I.

k=wave number $= \dfrac{2\pi}{\lambda}$

$\kappa = \dfrac{\alpha c}{\omega}$=imaginary part of the complex index of refraction $\tilde{n}$

Physically this captures a property of the material that the light is going into, in particular, how much the material attenuates the wave. Specifically

$$A(x) = A(0)e^{-\alpha x}.$$

Let's focus on the real part of the complex index of refraction, the n. What is this for a material? In practice, this slows the speed of the wave:

$$v_{\text{medium}} = \frac{c}{n}$$

Suppose we had an electric field…

II. Recall the wave equation

$$\frac{\partial^2 E(x,t)}{\partial t^2} = c^2 \frac{\partial^2 E(x,t)}{\partial x^2}$$

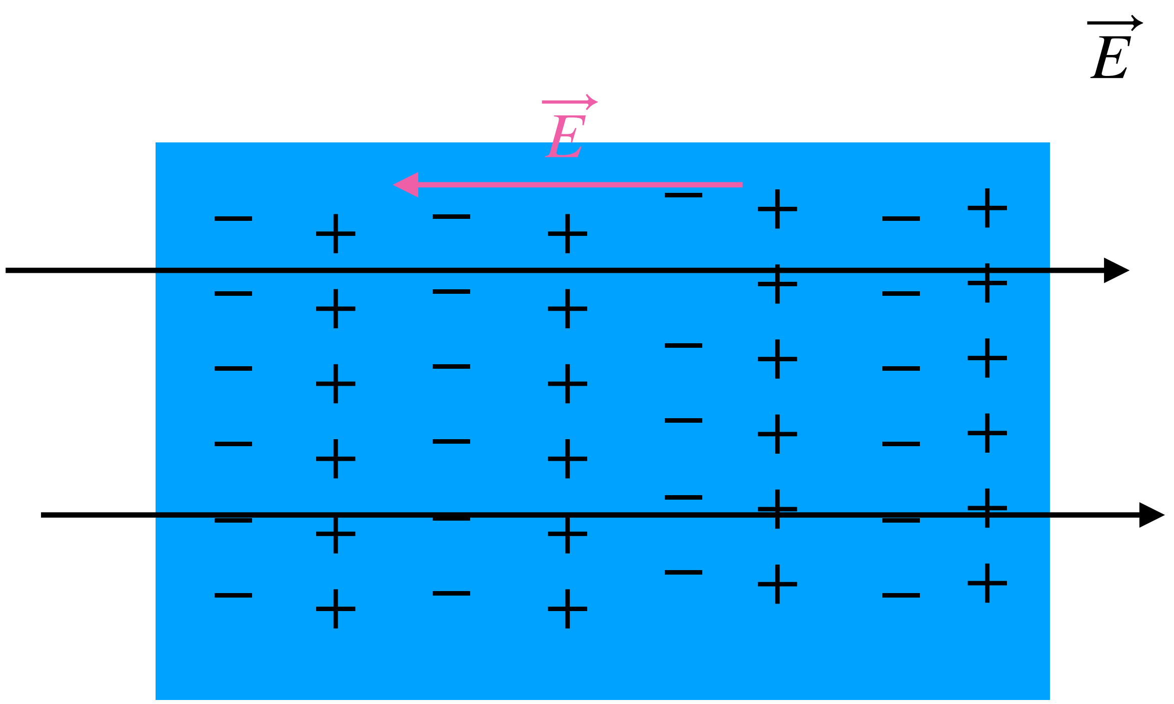
If you derive this wave equation from Maxwell's equations for the Electric and Magnetic fields you find

$$\frac{\partial^2 E(x,t)}{\partial t^2} = \frac{1}{\mu_0 \epsilon_0} \frac{\partial^2 E(x,t)}{\partial x^2}.$$

Here μ_0 is the permeability of free space and ϵ_0 is permittivity of free space. Then $c = 1/\sqrt{\epsilon_0 \mu_0} = 3 \times 10^8 \mathrm{m/s}$.

In a material we use the new permittivity: $\epsilon_r \epsilon_0$, here ϵ_r is the relative permittivity and is a pure number. We also have $\mu_r \mu_0$, interestingly then

$$v_{\text{medium}} = \frac{1}{\sqrt{\epsilon_r \epsilon_0 \mu_r \mu_0}} = \frac{c}{\sqrt{\epsilon_r \mu_r}} \equiv \frac{c}{n}$$

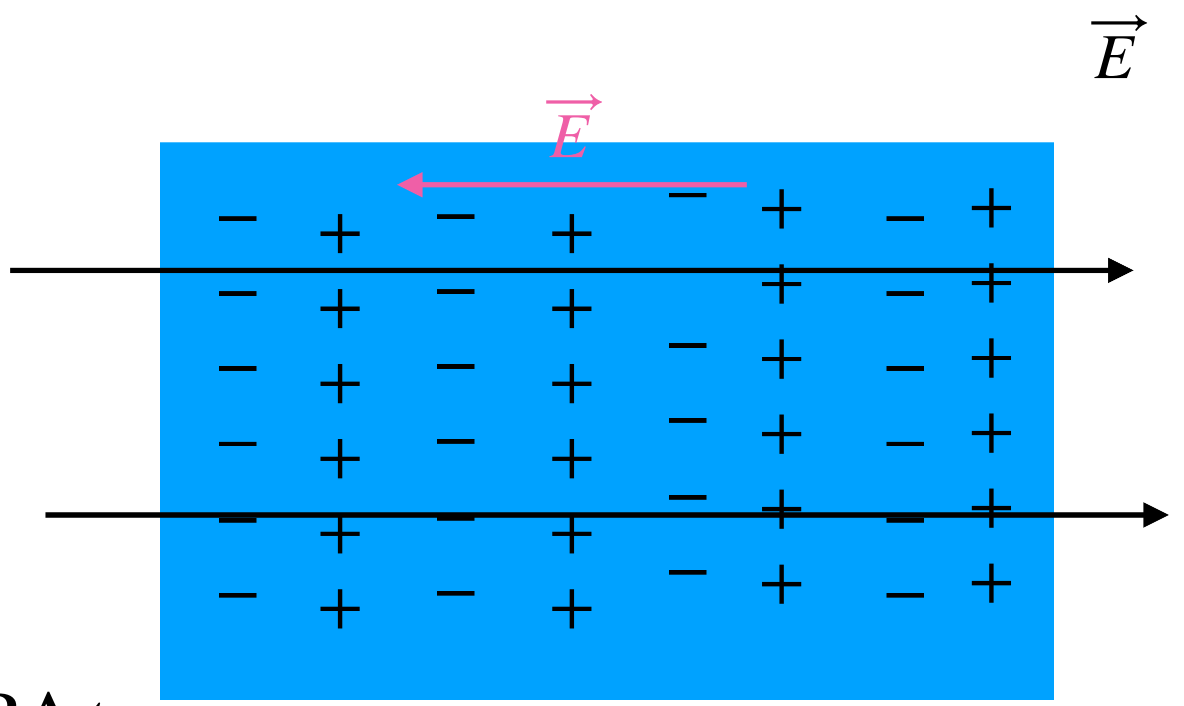


II.

For most materials we have $\mu_r \approx 1$. So really, the index of refraction is basically a measure of the permittivity of a medium:

$$v_{\text{medium}} = \frac{1}{\sqrt{\epsilon_r \epsilon_0 \mu_r \mu_0}} = \frac{c}{\sqrt{\epsilon_r \mu_r}} \approx \frac{c}{\sqrt{\epsilon_r}} \equiv \frac{c}{n}.$$

IV. Let's connect the random walk that you studied on the homework to the diffusion equation: we've got a one dimensional line and because the walkers take discrete steps, they can only be located at discrete positions along the line, call the number of them at position i, $N_i(t)$. Let's call the probability that a walker leaves its current location $p = R\Delta t$



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Then the number walkers that go from position i to position $i + 1$,
is pN_i in time Δt. In total we have
$$N_i(t + \Delta t) = N_i(t) - R\Delta tN_i(t) - R\Delta tN_i(t) + R\Delta tN_{i-1}(t) + R\Delta tN_{i+1}(t)$$