Today

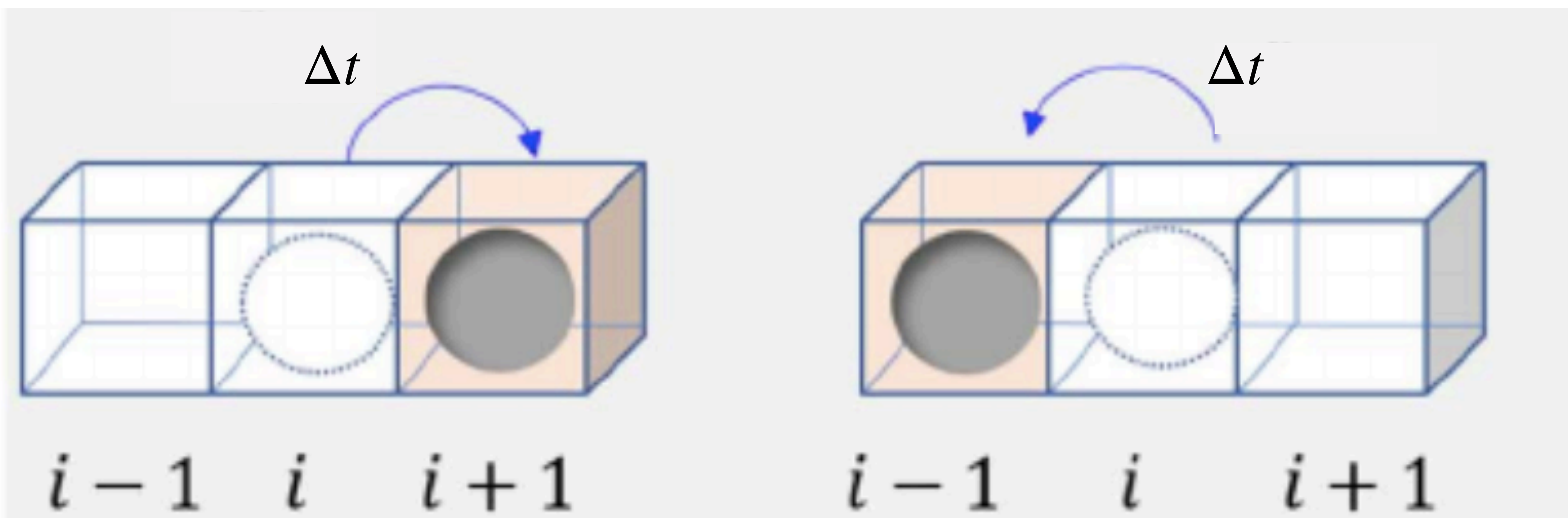
I. Last Time
II. Deriving the Diffusion Equation
III. Where do waves come from?

I. Yanpei Deng this week due to exam (will help in the lab. She's available MW from 7-8pm in Brody lab).

This week: Antu Antu will be providing homework support. Hours are: Tu 8-9pm, Th10:30-11:30am, Th 8-9pm.

Last time we discussed how the permittivity and permeability enter the wave equation for light:
$c = 1/\sqrt{\epsilon_0 \mu_0}$ (light speed in vacuum) or $v = c/n \approx c/\sqrt{\epsilon_r}$.

I. Let's connect the random walk that you studied on the homework to the diffusion equation: we've got a one dimensional line and because the walkers take discrete steps, they can only be located at discrete positions along the line, call the number of them at position i, $N_i(t)$. Let's call the probability that a walker leaves its current location $p = R\Delta t$.

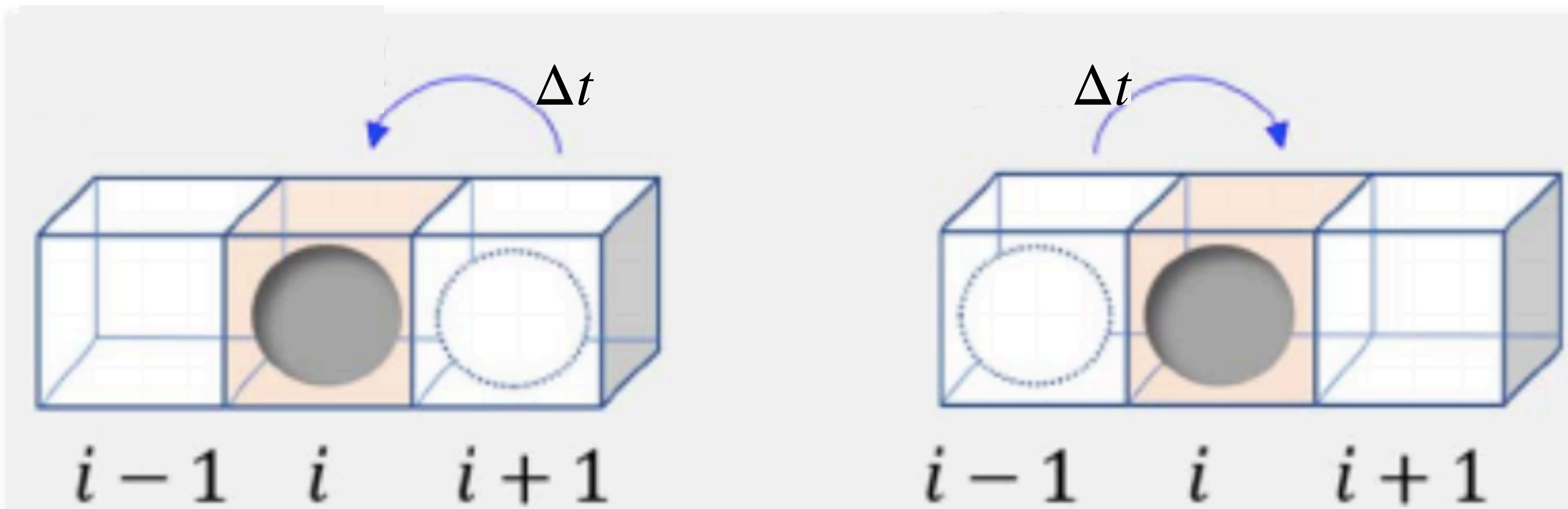


Then the number walkers that go from position i to position $i+1$, is pN_i in time Δt and the number that go to $i-1$ is pN_i in time Δt

$$N_i(t + \Delta t) = N_i(t) - R\Delta t N_i(t) - R\Delta t N_i(t) + \cdots$$

I. Meanwhile there are walkers that enter i from $i + 1$, and from $i - 1$. In total we have

$$N_i(t + \Delta t) = N_i(t) - R\Delta t N_i(t) - R\Delta t N_i(t) + R\Delta t N_{i-1}(t) + R\Delta t N_{i+1}(t).$$



Recall the definition of $f(x)$:

$$\frac{df}{dx} \equiv \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}.$$

Notice that

$$\frac{N_i(t + \Delta t) - N_i(t)}{\Delta t} = -RN_i(t) - RN_i(t) + RN_{i-1}(t) + RN_{i+1}(t)$$

II.

Notice that

$$\frac{N_i(t+\Delta t) - N_i(t)}{\Delta t} = R\Delta x^2 \frac{((N_{i+1}(t) - N_i(t)) - (N_i(t) - N_{i-1}(t)))}{\Delta x^2}$$

$$= R\Delta x^2 \frac{(\frac{(N_{i+1}(t) - N_i(t))}{\Delta x} - \frac{(N_i(t) - N_{i-1}(t))}{\Delta x})}{\Delta x}$$

[N.B.: The notation is a bit asymmetric. We're treating t as the argument of the function $N(t)$ and i as label. We could think of i as another argument of the function $N(t, i)$.]

In the limit of small Δt and small Δx we get

$$\frac{\partial N}{\partial t} = R\Delta x^2 \frac{\partial^2 N}{\partial x^2} \equiv D \frac{\partial^2 N}{\partial x^2},$$

where the 2nd line defines the "diffusion constant" D. If you look in the literature this "diffusion equation" is often described in terms of the "concentration" of the molecules $c = N/V_{\text{box}}$.

II.

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where the 2nd line defines the "diffusion constant" D. If you look in the literature this "diffusion equation" is often described in terms of the "concentration" of the molecules $c \equiv N/V_{\text{box}}$. The diffusion equation, after division by V_{box}, is

$$\boxed{\frac{\partial c}{\partial t} = D\frac{\partial^2 c}{\partial x^2}.}$$

Let's transition into a deeper study of waves and waves in materials.

III. Where do waves come from? Are they really the result of discrete particle motion?

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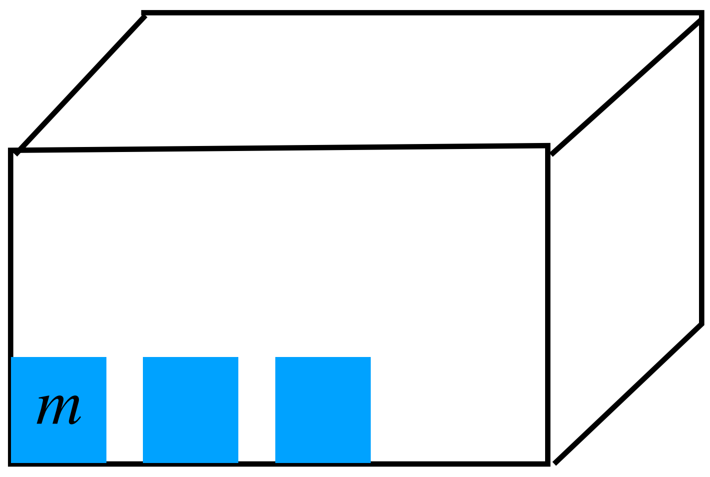
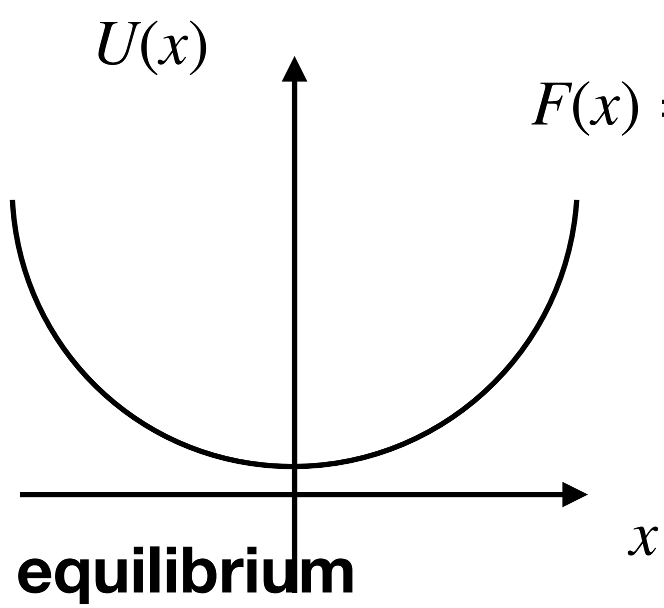
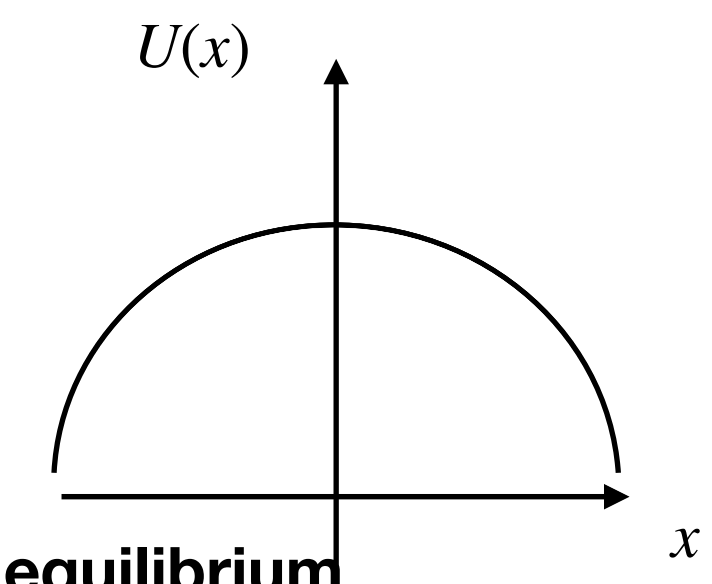


Usually a piece of solid is not moving. Then the constituent masses are at rest. For them to be at rest they must be at a stable equilibrium, that is, at a minimum of the potential energy



Stable equilibrium

$$F(x) = -\frac{dU}{dx}$$



Unstable equilibrium

III. In general, we can find equilibria by

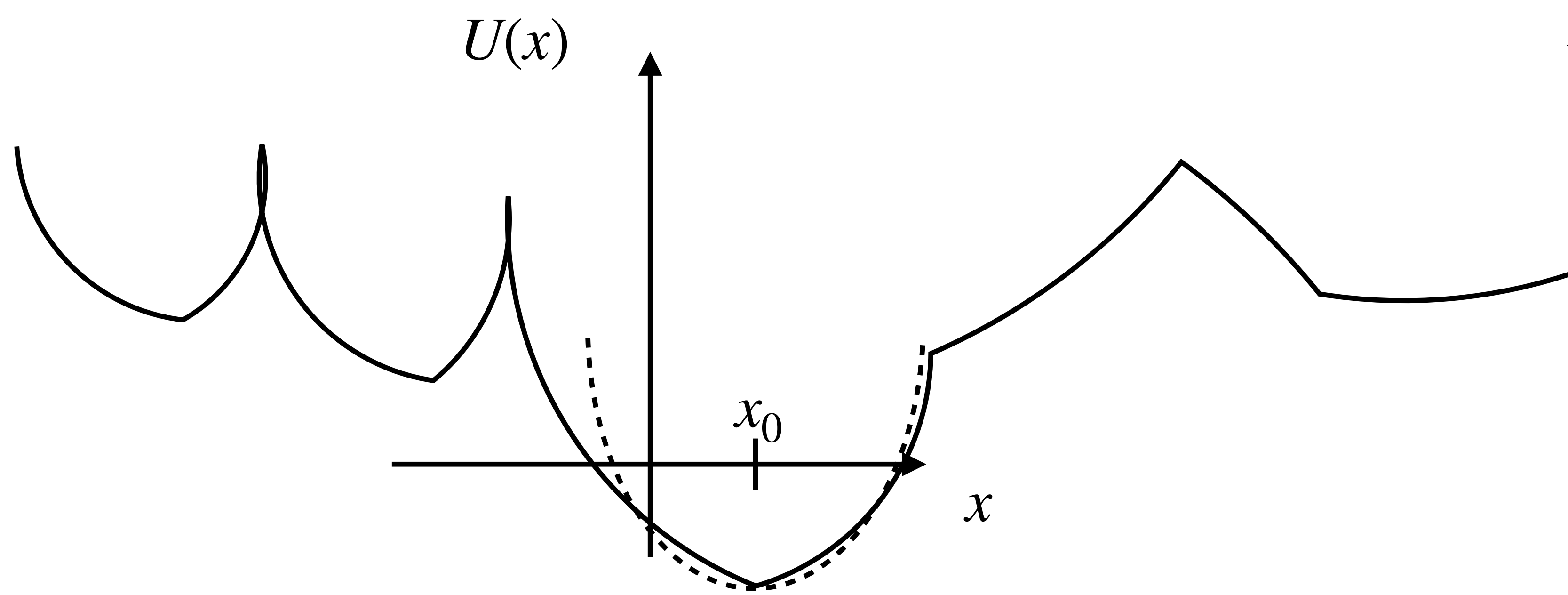
$$\frac{dU}{dx} = 0,$$

These equilibria will be stable if

$$\frac{d^2U}{dx^2} > 0, \text{ and unstable if } \frac{d^2U}{dx^2} < 0.$$

Let's consider a completely general potential $U(x)$

$$F(x) = -\frac{dU}{dx}$$



We can expand $U(x)$ around x_0 using Taylor expansion:

$$U(x) = U(x_0) + U'(x_0)(x - x_0) + \frac{1}{2}U''(x_0)(x - x_0)^2 + \cdots + \frac{1}{n!}U^{(n)}(x_0)(x - x_0)^n \cdots$$

III. What's an example of all of this?

We can expand $U(x)$ around x_0 using Taylor expansion:

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Well, this should work for a harmonic oscillator:

$U(x) = \frac{1}{2}kx^2$, then

$$U(x) = \frac{1}{2}k(0)^2 + k(0)(x - 0) + \frac{1}{2}k(x - 0)^2 = \frac{1}{2}kx^2.$$

In the real world a potential only links "harmonic" , that is, like a spring for part of the range of its x variable.

To calculate the "spring constant" of a harmonic potential , all I need to do is compute $k = \left.\frac{d^2U}{dx^2}\right|_{x_0}$.