Today

I. Days that we'll be taking a break: A week from this Friday, Oct 23rd there will be no class or homework meetings, Wed Nov 25th also no class, or Fri Nov 27th.
II. Last Time
III. Derivation of the Wave Equation

I. Yanpei Deng this week due to exam (will help in the lab. She's available MW from 7-8pm in Brody lab).

This week: Antu Antu will be providing homework support. Hours are: Tu 8-9pm, Th10:30-11:30am, Th 8-9pm.

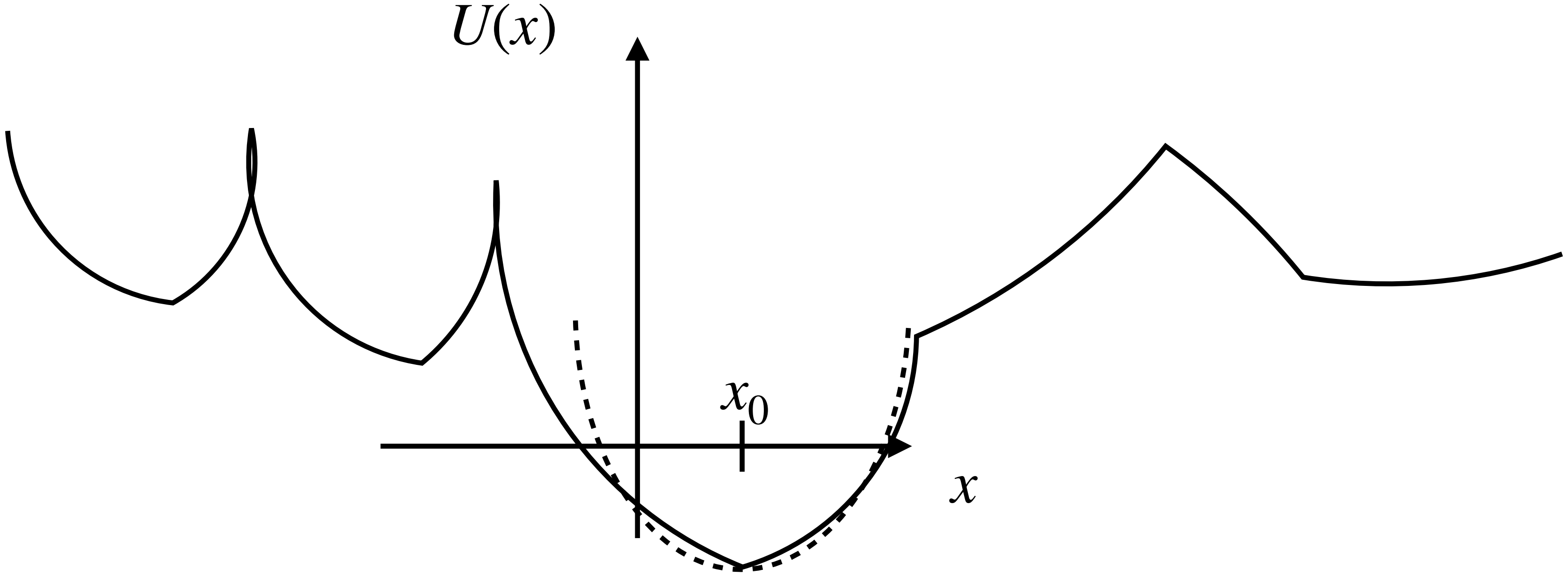
II. We discussed and derived the diffusion equation.

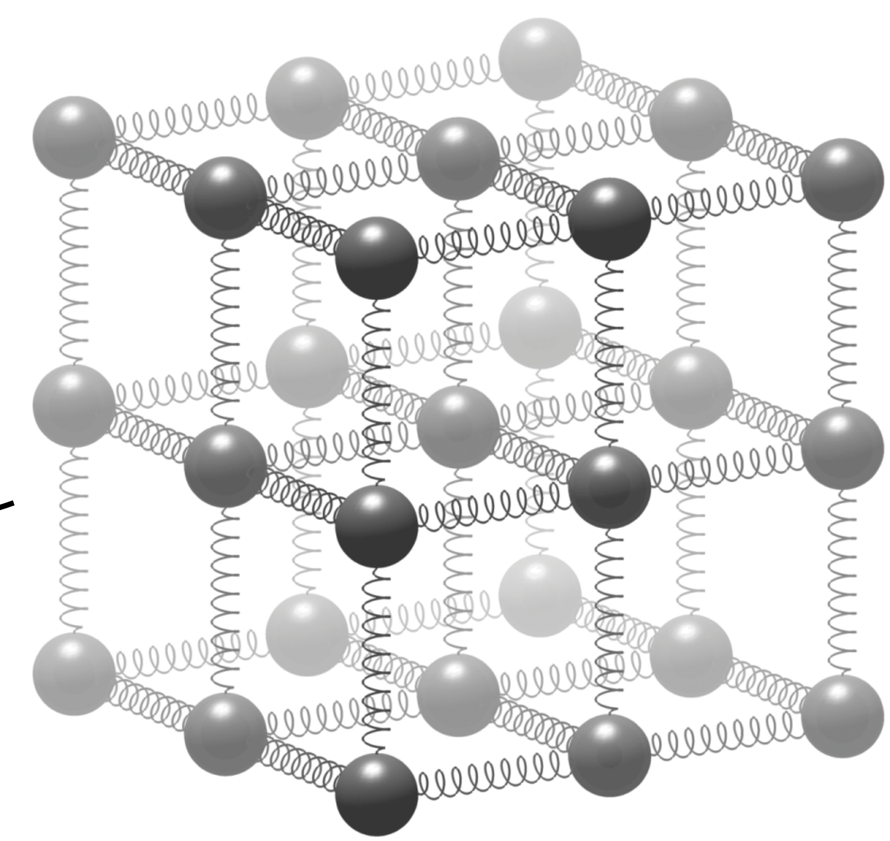
$\frac{\partial c}{\partial t} = D\frac{\partial^2 c}{\partial x^2}$, where c is the concentration $c = N_i/V_{\text{box}}$ and D is the "diffusion constant" with units meters squared per second.

I. We reviewed Taylor expansion in the context of potential energies and, in particular, around a stable equilibrium: near a stable equilibrium we can approximate any system whatsoever as being nearly a harmonic oscillator!

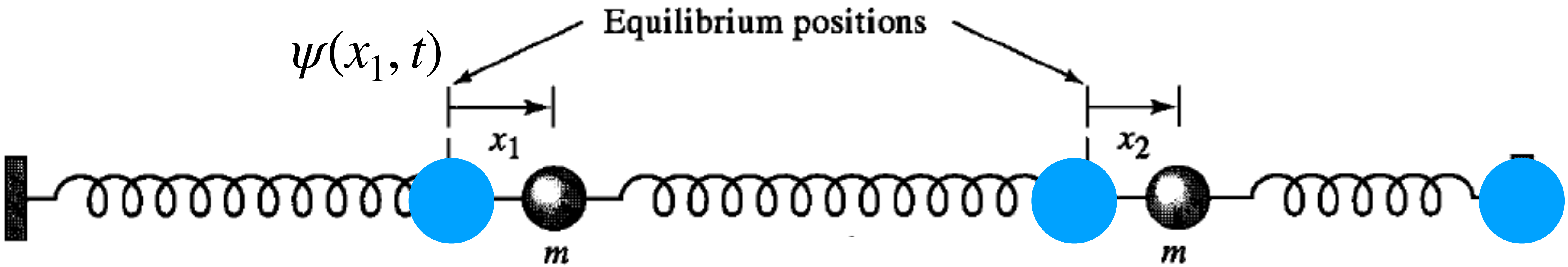
A chunk of material on the table is certainly in a stable equilibrium. Then all of its constituents can be modeled as if they were connected by springs. In a real material these forces will be electrical, but I can still model them as springs.

$$F(x) = -\frac{dU}{dx}$$





III. Consider a massive rod of length L and mass M. We'll model it as a collection of N masses, each one of mass m, connected via massless identical springs of spring constant k. In equilibrium our masses will be equally spaced with separation d.



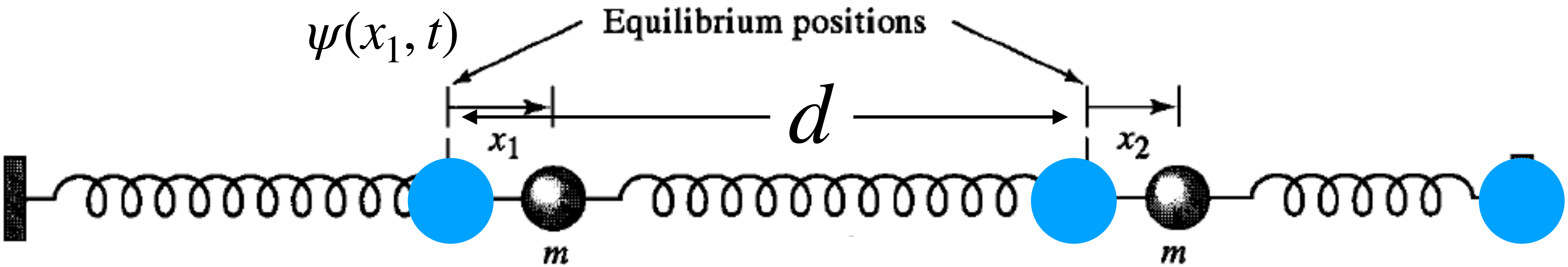
Let's call $\psi(x_n, t)$ the displacement of the mass at position x_n in the chain. We'll analyze this chain using Newton's 2nd law. Let's first analyze what the left spring does:

$F_L = -k[\psi(x_2, t) - \psi(x_1, t)]$ or in general $F_L = -k[\psi(x_n, t) - \psi(x_{n-1}, t)]$

$F_R = k[\psi(x_3, t) - \psi(x_2, t)]$ or in general $F_R = k[\psi(x_{n+1}, t) - \psi(x_n, t)]$

Total:

$$F = k[\psi(x_{n+1}, t) - \psi(x_n, t) - (\psi(x_n, t) - \psi(x_{n-1}, t))] = ma_n = m\frac{\partial^2 \psi(x_n, t)}{}$$



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Total:

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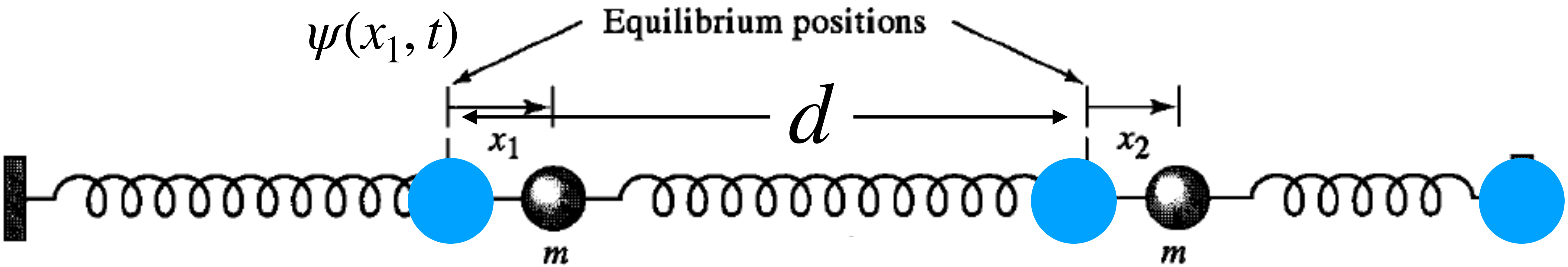
$$d^2 k \frac{\left[\frac{\psi(x_{n+1}, t) - \psi(x_n, t)}{d} - \frac{(\psi(x_n, t) - \psi(x_{n-1}, t))}{d}\right]}{d} = ma_n = m\frac{\partial^2 \psi(x_n, t)}{\partial t^2}$$



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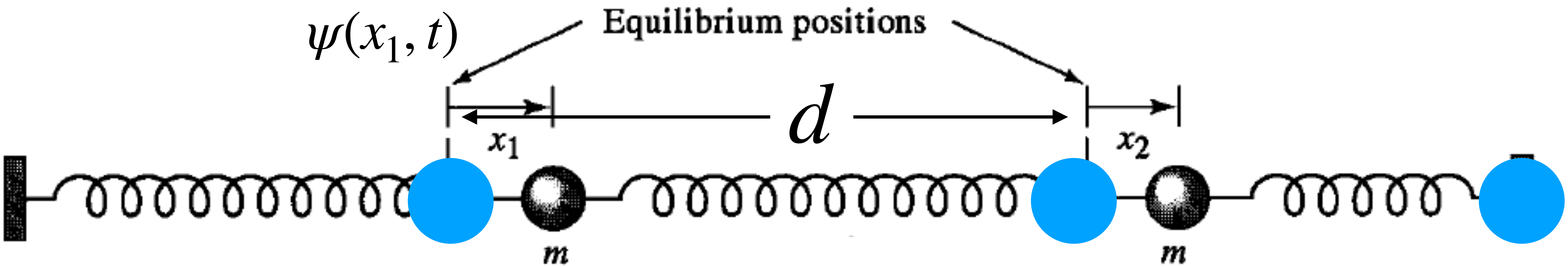
$$d^2k\frac{[\frac{\psi(x_{n+1},t)-\psi(x_n,t)}{d} - \frac{(\psi(x_n,t)-\psi(x_{n-1},t))}{d}]}{d} = ma_n = m\frac{\partial^2\psi(x_n,t)}{\partial t^2}$$

Before we can take any limits, we have to organize things carefully. Recall that in the setup our rod had a length L and N masses, then $d = \frac{L}{N}$. We also had M, so that $m = \frac{M}{N}$. This means that $\frac{d^2k}{m} = \frac{L}{N}\frac{d \cdot k}{\frac{M}{N}} = \frac{L}{M}d \cdot k = \frac{L^2}{M}\frac{k}{N} = \frac{L^2}{M}K$, where K is the "spring constant of the whole rod" (from addition of springs in series).



$$F = k[\psi(x_{n+1}, t) - \psi(x_n, t) - (\psi(x_n, t) - \psi(x_{n-1}, t))] = ma_n = m\frac{\partial^2 \psi(x_n, t)}{\partial t^2}$$

$$d^2 k \frac{[\frac{\psi(x_{n+1}, t) - \psi(x_n, t)}{d} - \frac{(\psi(x_n, t) - \psi(x_{n-1}, t))}{d}]}{d} = ma_n = m\frac{\partial^2 \psi(x_n, t)}{\partial t^2}$$

This means that

$$\frac{d^2 k}{m} = \frac{L}{N}\frac{d \cdot k}{\frac{M}{N}} = \frac{L}{M} d \cdot k = \frac{L^2}{M}\frac{k}{N} = \frac{L^2}{M} K.$$

Putting this all together and taking the limit $\lim_{d \to 0}$ gives

$\frac{L^2}{M} K \frac{\partial^2 \psi(x, t)}{\partial x^2} = \frac{\partial^2 \psi(x, t)}{\partial t^2}$. The speed is $c = L\sqrt{\frac{K}{M}}$.