

Today

- I. Upcoming exam, this Friday, Nov 6th.
- II. Last Time
- III. Heat Capacity of Ideal Gases
- IV. Adiabatic Expansion of an Ideal Gas

I. Yanpei Deng this week due to exam (will help in the lab. She's available M from 7-8pm in Brody lab).

This week: Antu Antu will be providing homework support. Hours are: Tu 8-9pm, Th 8-9pm.

II. We discussed the example of unfair footrace to illustrate inexact differentials:

1. The integral of an inexact differential doesn't just depend on the endpoints. Instead you have to give information about the entire history of how you got between the endpoints.

(Some quantities are process dependent, some quantities are not “state functions”, etc.)

2. For exact differentials we have the fundamental theorem of calculus (FTIC)

$$\int_{x_i}^{x_f} dx = x_f - x_i.$$

For inexact differentials we have no such result.

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$$c_{\text{water}} = 1 \frac{\text{cal}}{\text{g}^\circ\text{C}}.$$

In terms of more familiar units $1 \text{ cal} = 4.184 \text{ J}$.

III. Let's consider the heat capacity of gases. First let's take our gas to have a constant volume:

$$C = \frac{Q}{\Delta T} = \frac{\Delta U - W}{\Delta T} = \frac{\Delta U}{\Delta T}.$$

In the limit of very small changes we have

$$C_V = \left(\frac{\partial U}{\partial T} \right)_V.$$

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Let's predict the heat capacity at constant volume:

$$C_V = \left(\frac{\partial U}{\partial T} \right)_V = \frac{3}{2}Nk = \frac{3}{2}nR.$$

More common in the laboratory is to fix the pressure of a gas:

$$C_P = \frac{Q}{\Delta T} = \frac{\Delta U - W}{\Delta T} = \frac{\Delta U + P\Delta V}{\Delta T}.$$

Again taking the limit gives: $C_P = \left(\frac{\partial U}{\partial T} \right)_P + P \left(\frac{\partial V}{\partial T} \right)_P$

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Applying this to an ideal gas gives ($PV = NkT$ or $V = NkT/P$):

$$C_P = \frac{3}{2}Nk + P \frac{Nk}{P} = \frac{5}{2}Nk = \frac{5}{2}nR.$$

IV. Adiabatic expansion of ideal gases: coolers or any form of insulation prevents the exchange of heat between what's inside and what's outside. The point is to get no heat exchange!

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Let's consider an expansion of an idea gas, but in the very special circumstances where no heat flows in or out. How are pressure and volume related in these circumstances?

Again, the internal energy is $U = \frac{f}{2}NkT$, then

$$dU = \frac{f}{2}NkdT .$$

According to the first law of thermodynamics

$$dU = Q + W = W = -PdV.$$

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Setting these equal gives...

$$\frac{f}{2}NkdT = -PdV = -\frac{NkT}{V}dV.$$

We can cancel Nk , and divide by T to get

$$\frac{f}{2} \frac{1}{T} dT = -\frac{1}{V} dV.$$

Integrating both sides gives

$$\frac{f}{2}(\ln T_f - \ln T_i) = -(\ln V_f - \ln V_i) \text{ or } \ln \frac{T_f^{f/2}}{T_i^{f/2}} = \ln \frac{V_i}{V_f}.$$

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Can we express this as a relationship between P and V ? From the ideal gas law we have $PV = NkT$ or $T = PV/(Nk)$ and plugging this in gives

$V(PV)^{f/2} = V^{1+f/2} P^{f/2} = \text{const}$ or taking the $2/f$ th root we have

$$PV^{2/f+1} = PV^{\frac{2+f}{f}} \equiv PV^\gamma = \text{const},$$

where $\gamma \equiv (f+2)/f$. This allows us to make a direct comparison with the isothermal process, we have

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$$W = - \int P dV$$

