

# Today

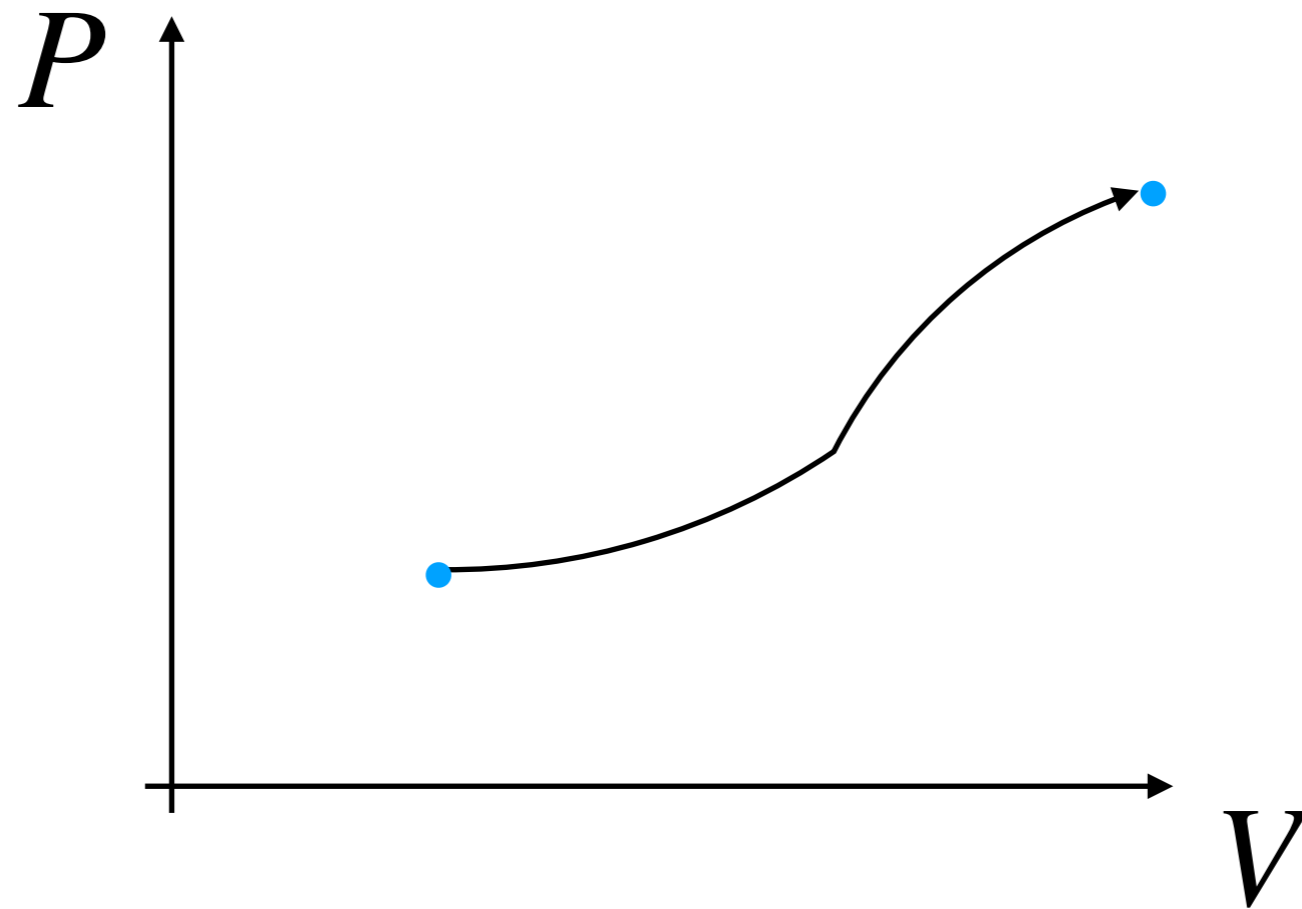
- I. Last Time
- II. Entropy as a State Variable
- III. Examples of Computing Entropy in Thermodynamics

I. We made a distinction between reversible and irreversible processes: a reversible process is one where you can retrace your steps, concretely we can imagine a piston that slowly compresses a gas and then slowly expands it back out.

An irreversible is one that creates entropy and cannot be easily reversed. Lighting an object on fire is a good example.

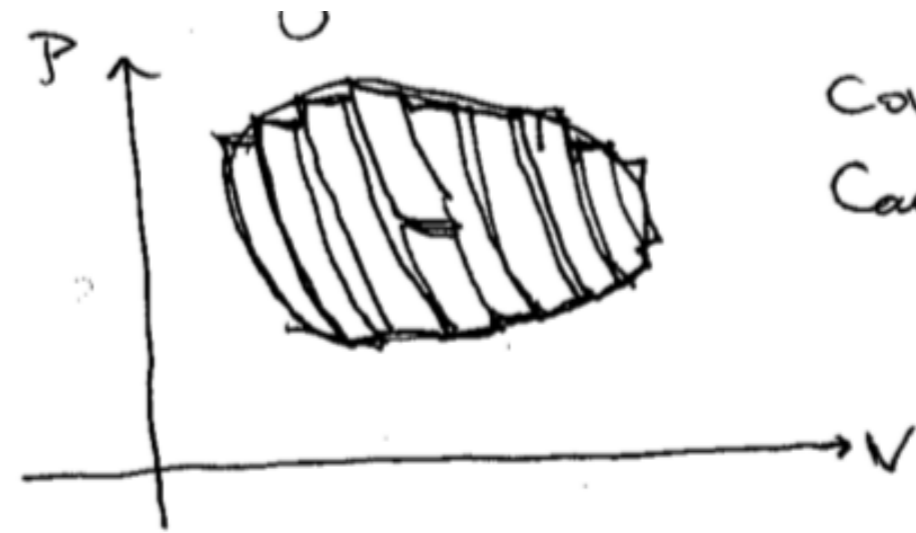
What role does entropy play in the behavior of engines?

I. What does what we learned about Carnot engines mean for reversible processes in general?



It's hard to tell from the diagram alone if a process was done reversibly or not.

II. There are three steps in our proof that Entropy is a state variable: Step 1



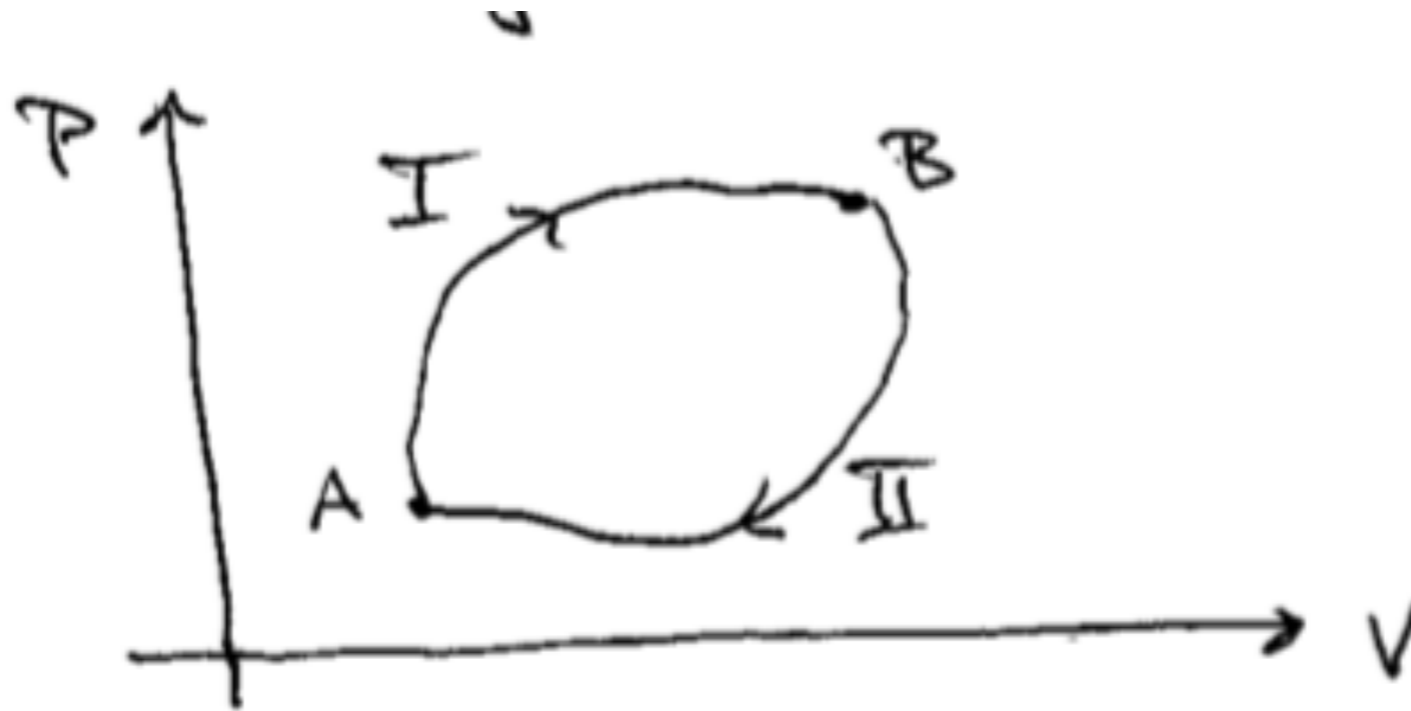
Cover it with Carnot cycles

$$\text{Step 1: } \oint \frac{\delta Q}{T} = 0$$

$$\text{Step 2: } \oint \frac{dQ}{T} = 0.$$

In Step 3 we will break open the closed cycle to see what we can say in general.

II. Consider a completely general reversible closed loop



According to step 2, we have

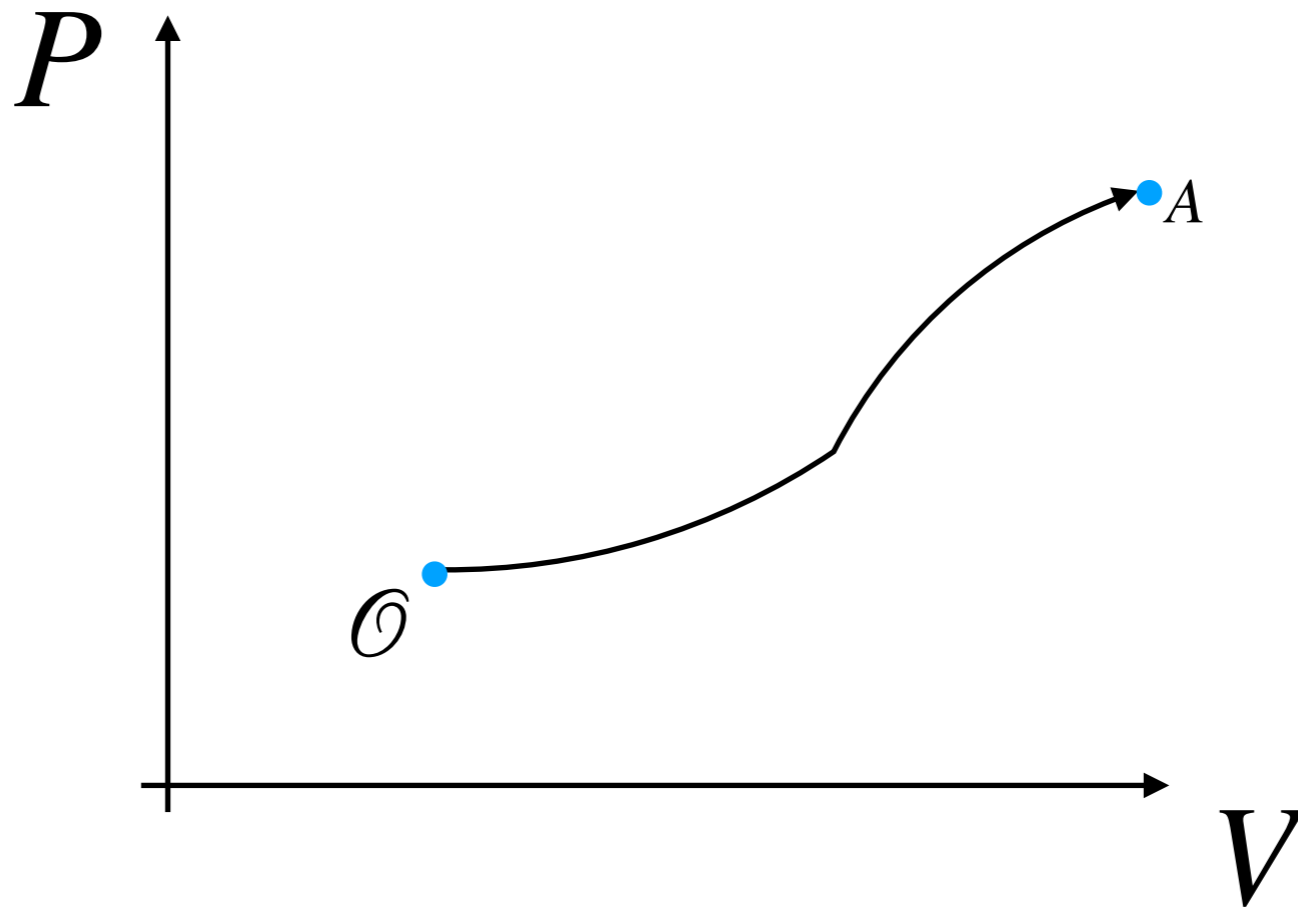
$$0 = \oint \frac{dQ}{T} = \int_{IA}^B \frac{dQ}{T} + \int_{IIB}^A \frac{dQ}{T} = \int_{IA}^B \frac{dQ}{T} - \int_{IIA}^B \frac{dQ}{T}$$

this then means that

$$\int_{IA}^B \frac{dQ}{T} = \int_{IIA}^B \frac{dQ}{T},$$
 since the loop was totally general this shows that

the result is completely independent of the path.

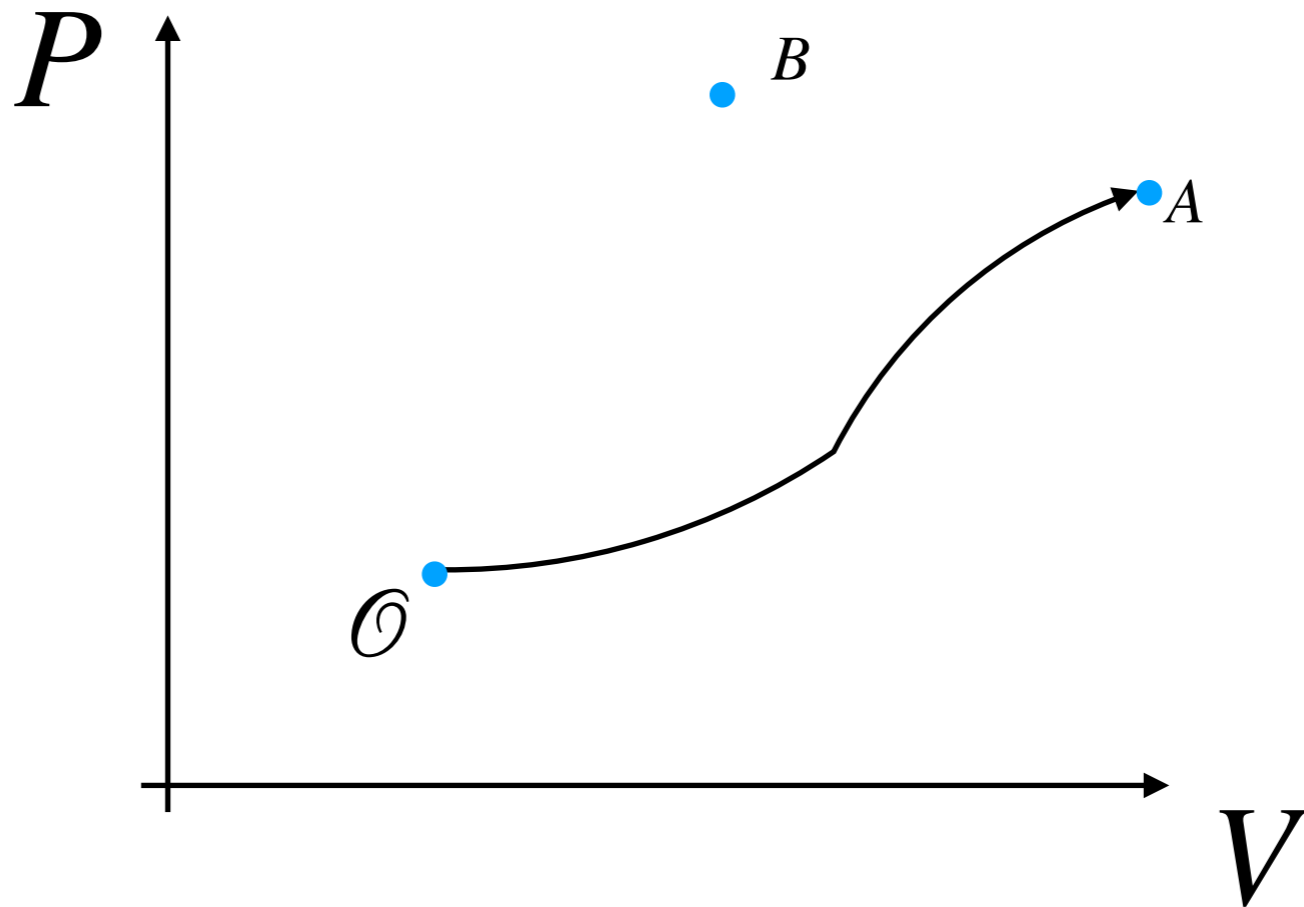
II. What we've just prove is that the ratio  $\delta Q/T$  is an exact differential! This means that there is some function, a state variable, such  $dS = \delta Q/T$ . This function is what we mean by the entropy  $S$ . To define  $S$  as a function we need to say where we start and where we go:  $S(A) = \int_{\mathcal{O}}^A \frac{\delta Q}{T}$ , this is called “the entropy of the system in state  $A$ ” (relative to  $\mathcal{O}$ ).



II. We can always think about entropy difference of two states:

$$\Delta S = S(B) - S(A) = \int_{\mathcal{O}}^B \frac{dQ}{T} - \int_{\mathcal{O}}^A \frac{dQ}{T} = \int_A^B \frac{dQ}{T},$$

This proves that the difference in entropy between any two states is independent of the reference point  $\mathcal{O}$ .



III. Examples: 1 kg of ice melts, what's  $\Delta S$ ? (Must be done at 0C in order for it to be reversible).

$$\Delta S = \int \frac{dQ}{T} = \frac{1}{T} \int dQ = \frac{Q}{T} = \frac{Lm}{T},$$

Here  $L$  is the specific latent heat of transformation. Looking this up in a table we find  $333 \times 10^3$  J/kg,

$$\Delta S = \frac{1}{273 \text{ K}} (333 \times 10^3 \text{ J/kg} \cdot 1 \text{ kg}) = 1.22 \times 10^3 \text{ J/K}.$$

Now, suppose we heat 1 kg of water from 0C to 100C,  $\Delta S=?$

$$\Delta S = \int \frac{dQ}{T} = \int \frac{mcdT}{T} = mc \ln \left( \frac{T_f}{T_i} \right) = (10^3 \text{ g})(4.19 \text{ J/g K}) \ln \left( \frac{373}{273} \right) = 1.31 \times 10^3 \text{ J/K}$$