Today

I. There will be (optional) class Monday, Dec 14th

II. Last Time

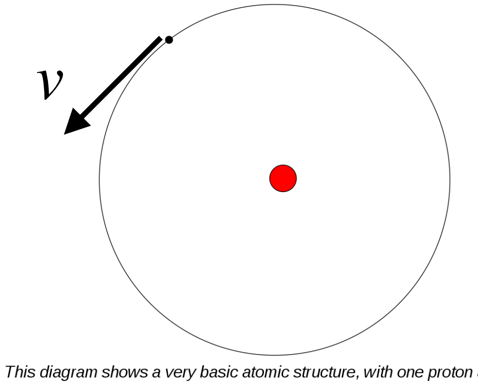
III. Bohr's Model of the Hydrogen Atom

I. Black body, Einstein's photoelectric explanation:

Both led to "waves as particles": $E = hf = h\nu$

De Broglie "particles as waves": $\lambda = \frac{h}{p}$ (here p is momentum)

Bohr called this wave-particle duality.



This diagram shows a very basic atomic structure, with one proton and one electron. This should depict the basic structure of the Hydrogen atom (H). Hydrogen has an atomic weight of 1.

II. Bohr's Model for Hydrogen (1913)

Hydrogen is the simplest atom: one proton and one electron

Classical part:

Coulomb's law: $F = \frac{1}{4\pi\epsilon_0}\frac{e^2}{r^2}$

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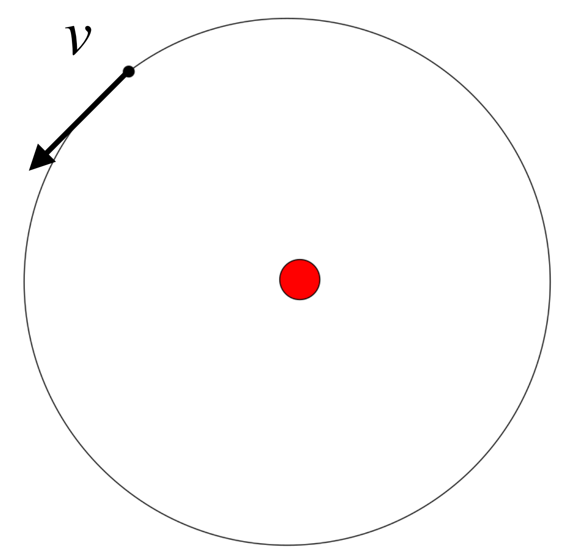
Coulomb's law: $F = \frac{1}{4\pi\epsilon_0}\frac{e^2}{r^2}$

Newton's 2nd law: $F = ma$

Centripetal acceleration: $a = \frac{v^2}{r}$



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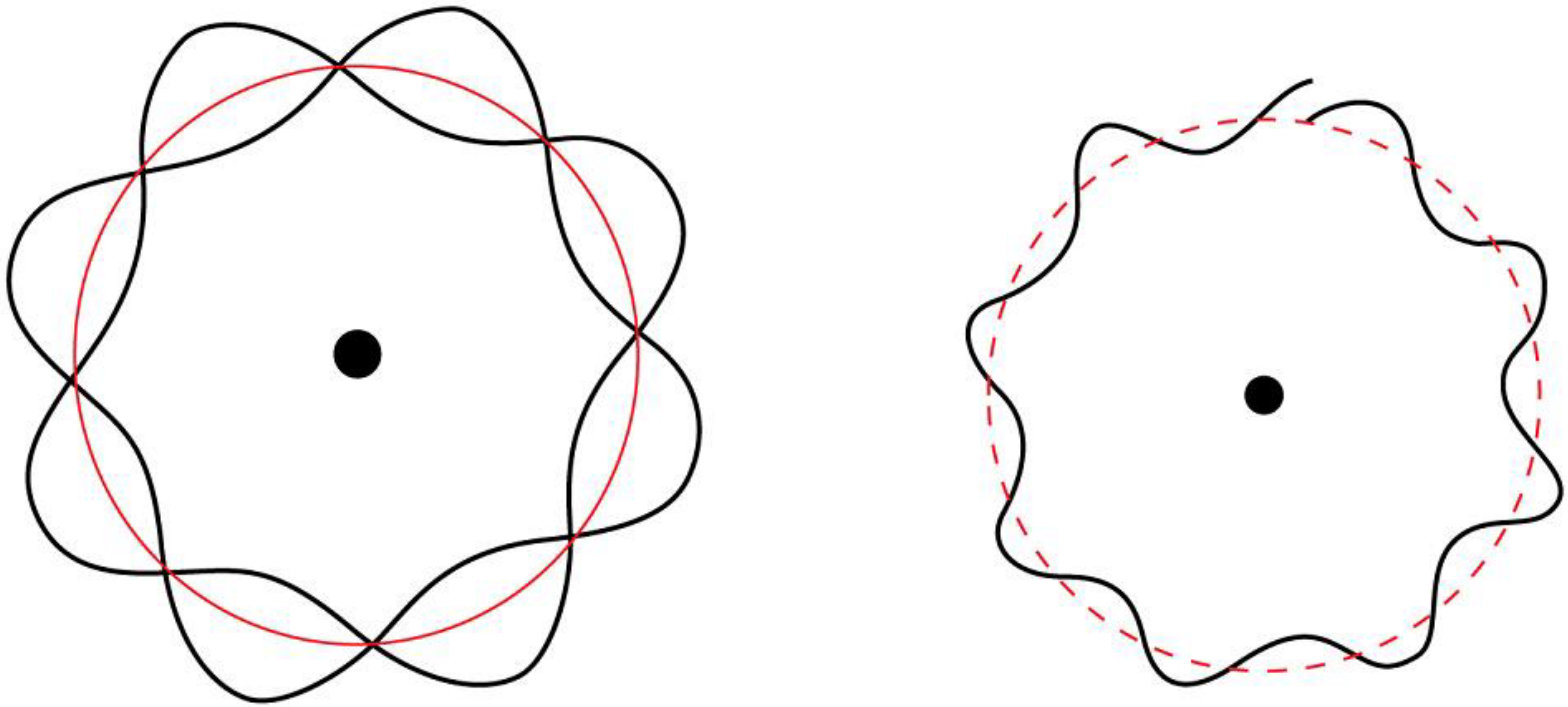
$$\frac{1}{4\pi\epsilon_0}\frac{e^2}{r^2} = \frac{mv^2}{r} \implies mv^2 = \frac{1}{4\pi\epsilon_0}\frac{e^2}{r} \quad (1)$$

Quantum Part: De Broglie: $\lambda = \frac{h}{p} = \frac{h}{mv}$

Divine inspiration: electron wave forms a standing wave pattern around the circumference of the orbit…

II. Electron orbits that constructively interfere over the whole orbit lead to stable motion in the orbit.



$2\pi r = n\lambda, \quad n = 1,2,3,\cdots$ (condition for constructive interference)

Putting in de Broglie

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Then, putting (2) into (1), solve for r:

$$m\frac{n^2h^2}{4\pi^2m^2r^2} = \frac{1}{4\pi\epsilon_0}\frac{e^2}{r} \quad \Longrightarrow \quad r_n = \left(\frac{h^2\epsilon_0}{\pi m e^2}\right)n^2, \quad n = 1,2,3,\cdots$$

Here the Bohr radius is $a \equiv \dfrac{h^2\epsilon_0}{\pi m e^2} = 0.529 \times 10^{-10}\mathrm{m}.$

<u>Energy levels:</u>

Returning to classical physics for a moment: $E = K.E. + P.E.$

$K.E. = \dfrac{1}{2}mv^2$, $P.E. = -\dfrac{1}{4\pi\epsilon_0}\dfrac{e^2}{r}$, then

$$E = \frac{1}{2}mv^2 - \frac{1}{4\pi\epsilon_0}\frac{e^2}{r} = \frac{1}{2}\frac{1}{4\pi\epsilon_0}\frac{e^2}{r} - \frac{1}{4\pi\epsilon_0}\frac{e^2}{r} = -\frac{1}{8\pi\epsilon_0}\frac{e^2}{r}$$

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Put in Bohr's formula for the allowed radii and you get:

$$E_n = -\frac{1}{8\pi\epsilon_0}\frac{e^2\pi m e^2}{h^2\epsilon_0 n^2} = -\frac{me^4}{8\epsilon_0^2h^2n^2} \equiv \frac{E_1}{n^2}, \; E_1 \equiv -\frac{me^4}{8\epsilon_0^2h^2} = -13.6\text{ eV}$$

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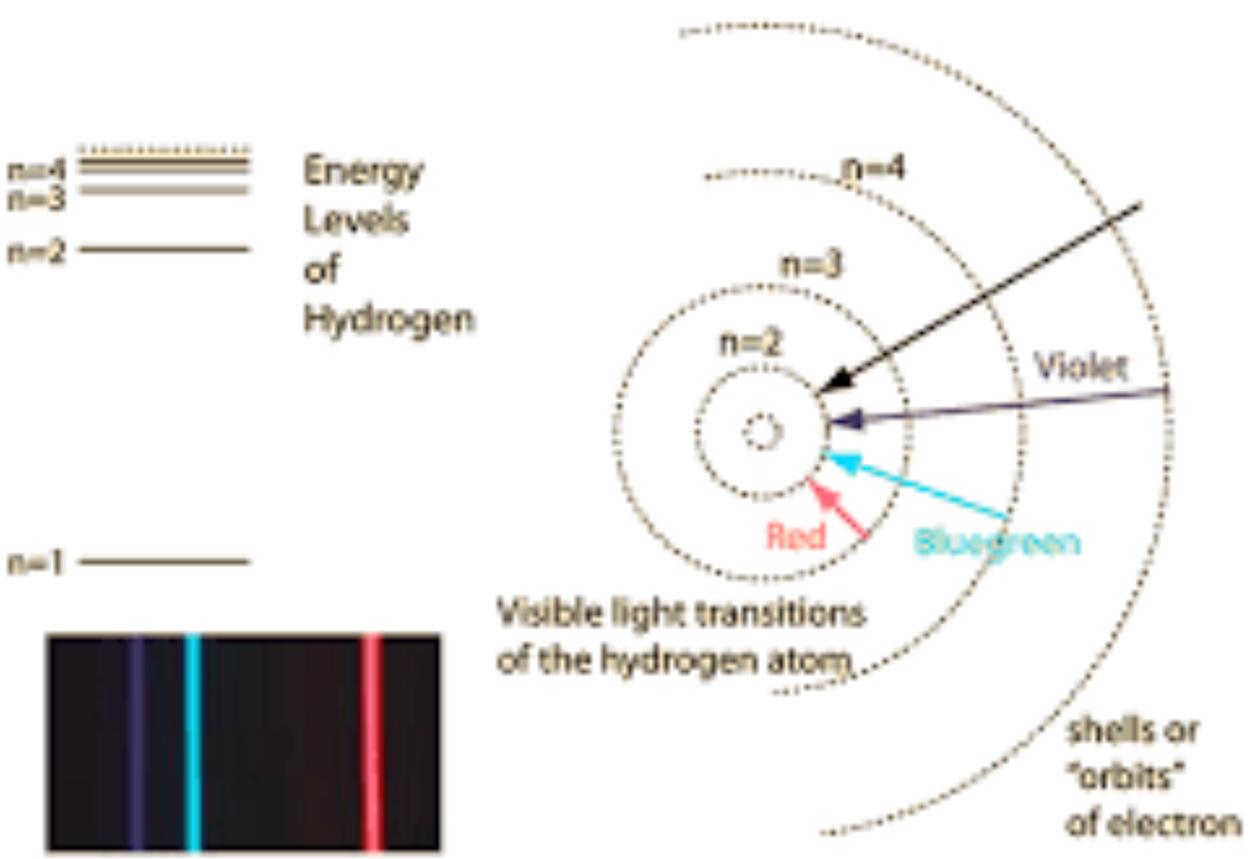

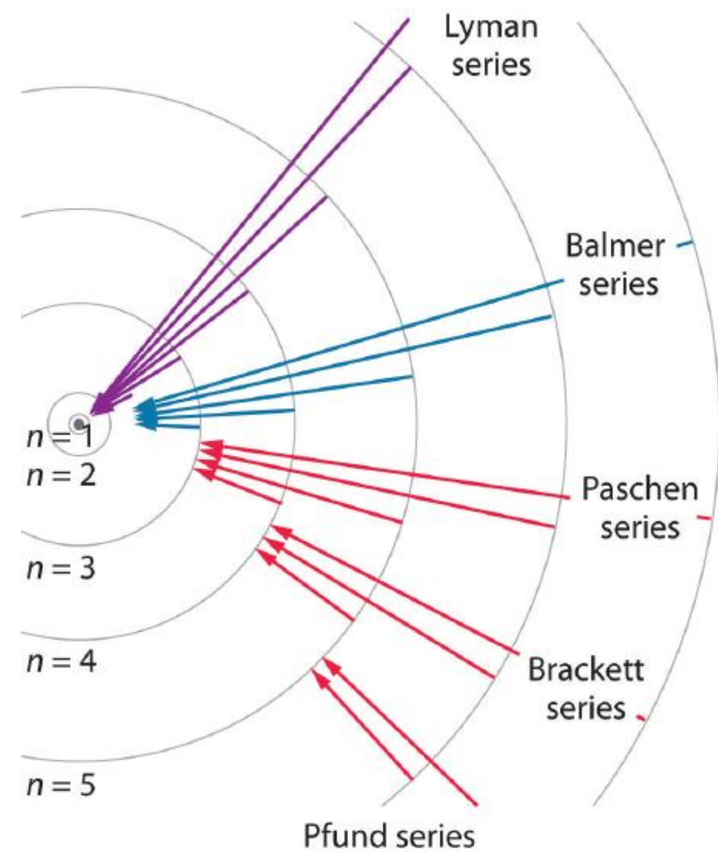

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In quantum theory, there are "quantum jumps" from higher energy levels to lower, and light is emitted during these jumps. It's that light that carries the difference in the energy:

Lyman series (U.V.) $n_f = 1$

Balmer series (visible) $n_f = 2$

Pashen series (infrared) $n_f = 3$

$$E_{\text{photon}} = E_i - E_f = \frac{E_1}{n_i^2} - \frac{E_1}{n_f^2} = hf = h\frac{c}{\lambda}, \text{ then}$$

$$\frac{1}{\lambda} = \frac{E_1}{hc}\left(\frac{1}{n_i^2} - \frac{1}{n_f^2}\right) = R_H\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right), R_H = \frac{me^4}{8\epsilon_0^2 h^3 c} = 1.097\times 10^7\text{m}^{-1}$$