Today

I. Last Time
II. Wrap up Discussion of Schroedinger Equation
III. The Infinite Square Well

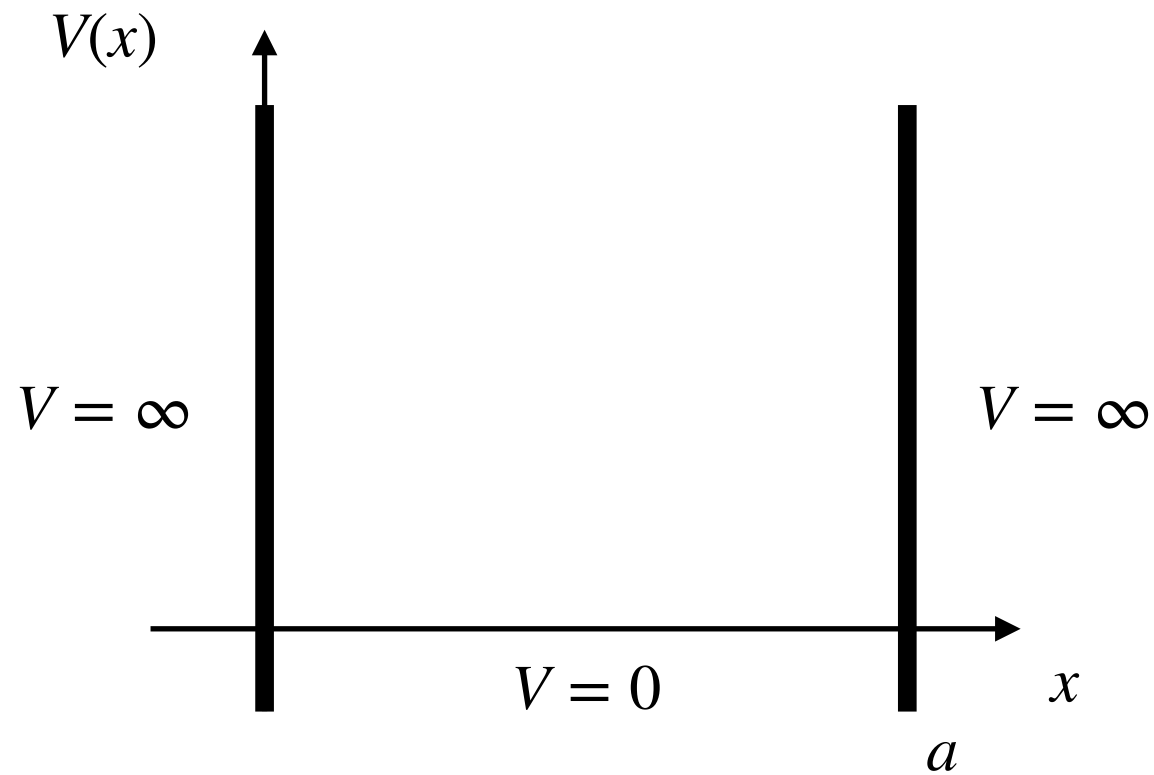
I. Introduced operators, which acted on the waves and returned physical values: e.g. $\hat{p} = \frac{\hbar}{i}\frac{\partial}{\partial x}$ (recall $[\hbar] =$ m kg m/s).

$$\hat{H}\Psi(x,t) = i\hbar\frac{\partial \Psi}{\partial t} = \frac{\hat{p}^2}{2m}\Psi + \widehat{V}(x)\Psi(x,t) = -\frac{\hbar^2}{2m}\frac{\partial^2 \Psi}{\partial x^2} + V(x)\Psi$$

This is the time-dependent Schroedinger equation. We also had

$-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} + V(x)\psi(x) = E\psi(x)$ (the time-indep. Sch. Eq.)

II. Today we're going to study an example called teh infinite square well:



As an equation we can write this in the following form:

$$V(x) = \begin{cases} 0, & \text{if } 0 \le x \le a, \\ \infty, & \text{otherwise}. \end{cases}.$$

For this potential we can focus our study of the Sch. Eq. to the interior of the box.

II. Where $V = 0$ we have

$$-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} = E\psi(x),$$

or

$$\frac{d^2\psi}{dx^2} = -\frac{2mE}{\hbar^2}\psi(x) = -k^2\psi(x). \quad \left(\text{Here } k \equiv \frac{\sqrt{2mE}}{\hbar}\right)$$

(Recall the harmonic oscillator had Newton's equation give by

$$\frac{d^2x}{dt^2} = -\omega^2 x.$$

With solutions $x(t) = A\cos(\omega t) + B\sin(\omega t)$.) Then

$$\psi(x) = A\sin(kx) + B\cos(kx). \text{ (check it!)}$$

In addition to the Sch. Eq. , we have boundary conditions which are

$$\psi(x=0) = 0 \quad \text{and} \quad \psi(x=a) = 0.$$

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In addition to the Sch. Eq. , we have boundary conditions which are

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Plugging in $x = 0$ gives,

$$\psi(0) = A\sin(0) + B\cos(0) = B = 0.$$

We must have $B = 0$. Then,

$$\psi(x) = A\sin(kx).$$

We still have the boundary condition at $x = a$,

$$\psi(a) = A\sin(ka) = 0.$$

Instead of fixing $A = 0$, which is boring, we choose

$$ka = n\pi, \quad n = 0, \pm 1, \pm 2, \pm 3, \cdots.$$

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Physically this amounts to a requirement of only particular energies:

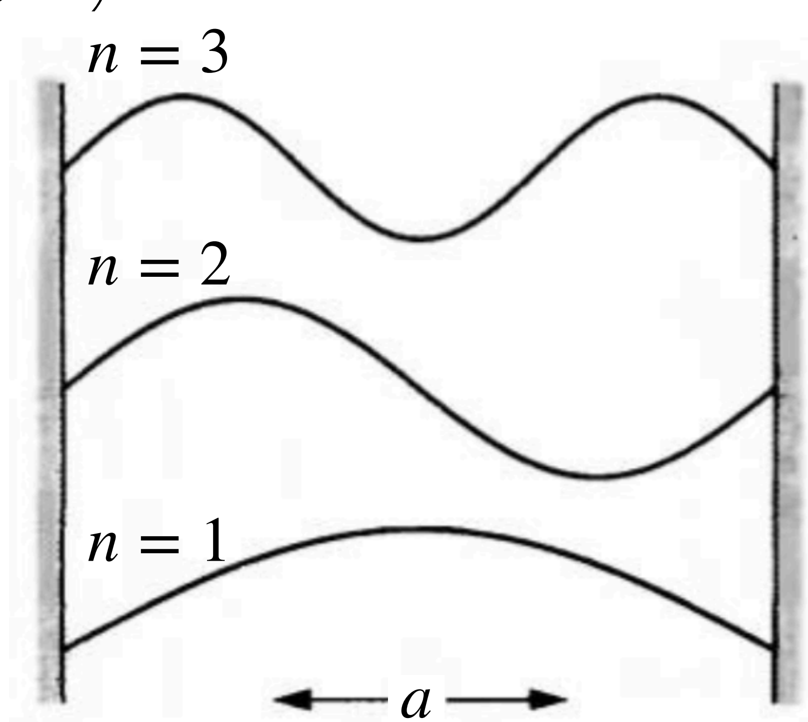
$$\frac{\sqrt{2mE_n}}{\hbar} = \frac{n\pi}{a}, \text{ with } n = 1,2,3,\cdots.$$

Solving for the energy we get

$$E_n = \frac{\hbar^2 k_n^2}{2m} = \frac{n^2\pi^2\hbar^2}{2ma^2}. \; (n = 1,2,3,\cdots)$$

Returning to our wave function we have

$$\psi(x) = A\sin\left(\frac{n\pi}{a}x\right), \; n = 1,2,3,\cdots.$$



$|\psi(x)|^2$ = probability density for particle position