Today

I. Announcements: Hal's office hours this week are T2-3pm, W 4:30-5:30pm, and Th 2:30-4pm.
II. Last Time
III. Your Questions
IV. Overview of Where We've Been
V. CAFE forms: https://tools.bard.edu/tools/cafeform/

II. We solve the full Sch. Eq. by first guessing solutions of the form

$$\Psi(x,t) = \psi(x)\varphi(t).$$

Little psi satisfies the time-independent Sch. Eq.

$$\hat{H}\psi = E\psi, \text{ or}$$

$$-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} + V(x)\psi = E\psi.$$

To do an example, we've chosen the infinite square well.

II. We found solutions of the form

$$\psi(x) = A\sin(kx) = \sqrt{\frac{2}{a}}\sin\left(\frac{n\pi}{a}x\right),\ n = 1,2,3,\cdots.$$

The fact that only certain k were allowed meant that only certain energies were allowed:

$$E_n = \frac{n^2\pi^2\hbar^2}{2ma^2}, \quad n = 1,2,3,\cdots.$$

We also found a completely general solution for little phi

$$\varphi(t) = e^{-iE_nt/\hbar}.$$

Having found all of this, we reassembled it into a total time-dependent solution:

$$\Psi_n(x,t) = \psi(x)\varphi(t) = \sqrt{\frac{2}{a}}\sin\left(\frac{n\pi}{a}x\right)e^{-iE_nt/\hbar}, \quad n = 1,2,3,\cdots.$$

Interpretation: $\int_b^c |\Psi(x,t)|^2\,dx$ is the probability of finding our particle between b and c at the time t.

III. A few more comments on boundary conditions:

For the infinite square well, we noticed that a particle that was below $x = 0$ or above $x = a$ would have an infinite energy/ Why? This was because of the infinite potential. Disallowing infinite energy led us to require zero wave function outside the well. Then we required that the wave function be continuous at the boundary, which amounts to requiring that $\psi(x = 0) = 0$ and $\psi(x = a) = 0$. (Technically, we also require that the derivative of the wave function be continuous everywhere except where the potential is infinite.)

When we think of space as being unbounded, we require that the wave function goes to zero as $x \to \pm\infty$.

III. How do we study quantum mechanics for different physical systems? We use different potentials in the Sch. Eq.

Harmonic oscillator: $V(x) = \frac{1}{2}kx^2 = \frac{1}{2}m\omega^2x^2$

$$-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} + V(x)\psi = E\psi \quad \text{or} \quad -\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} + \frac{1}{2}m\omega^2x^2\psi = E\psi$$

Infinite square well: $V(x) = 0$ for $0 < x < a$

$$-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} + V(x)\psi = E\psi \quad \text{or} \quad -\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} = E\psi$$

Gravity near Earth: $V(x) = mgx$

$$-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} + V(x)\psi = E\psi \quad \text{or} \quad -\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} + mgx\psi = E\psi$$

III. Give a force that I know $F(x)$ (e.g. Hooke's law $F = -kx$), when can I write that force in terms of a potential energy?

This is possible whenever the force is "conservative".

$$F = -\frac{dV}{dx}, \text{ solving for } V \text{ gives } V(x) = -\int_{\mathcal{O}}^{x} F dx.$$

A conservative force is one for which the final integral is independent of path, and hence defines a unique potential.