Homework 5

Due Friday, October 2nd in class

Start reading Chapter 8 of Taylor's Classical Mechanics.

1. Taylor 7.11, p282
2. Taylor 7.14, p283
3. Taylor 7.24 & 7.27, p285
4. Taylor 7.30, p286
5. Taylor 7.41, pp288 & 289
6. Consider our three-rod model for a "cat".

   (**a**) Begin with all three rods aligned along the α-axis. Calculate the change in orientation, $\Delta\theta$, of the whole system along the curve in shape space that runs from $\alpha = 0$ to $\alpha = 2\pi$ along the α-axis. Draw the cat before and after the traversal of the path. [Note that because shape space is a torus, this is a closed curve. What does this mean about the shape of the cat at the end of the path?]

   (**b**) Calculate $\Delta\theta$ for the same setup as in part (**a**) but use a path that runs along the β-axis from $\beta = 0$ to $\beta = 2\pi$. Draw the cat before and after the traversal of the path.

   (**c**) Calculate the "field strength,"

   $$B = \frac{\partial A_\beta}{\partial \alpha} - \frac{\partial A_\alpha}{\partial \beta}.$$

   (**d**) Now return to the four-legged path that we considered in class (a square of side length $\pi/2$ in shape space). Integrate the field strength B over this whole square:

   $$\int_0^{\pi/2}\int_0^{\pi/2} B d\alpha d\beta.$$

   Compare this result to the sum of all four $\Delta\theta$ that we calculated in class. If you have taken vector calculus explain why these two calculations compare the way that they do. If you haven't, simply write: "I haven't taken vector calculus" to get full credit.

   (**e**) Now that we are considering a more complicated three-rod model, the transformation to body axes can also be more complicated. For example, if we place the x_b' axis parallel to the third rod the transformation from θ to θ' will certainly depend on both α and β. In general, we can write this as

   $$\theta' = \theta + \psi(\alpha, \beta),$$

   where ψ is a function of both shape coordinates. Recall that we defined A_α and A_β as the coefficients of $\dot{\alpha}$ and $\dot{\beta}$ in the expression

   $$\dot{\theta} = A_\alpha \dot{\alpha} + A_\beta \dot{\beta}.$$

   If we switch to the new coordinate θ', what are the expressions for the new potentials A_α' and A_β' in terms of A_α, A_β and ψ?

   (**f**) Show that $B' = B$. This shows that the field strength is gauge invariant and partially explains its value. All of these calculations work similarly in E & M and other gauge theories.