Classical Mechanics
Day 1

Outline:

0. Review Newtonian Mechanics

I. Why are simple harmonic oscillators everywhere?

II The Standard Guess

Newtonian mechanics. Why?
All of you are missing different pieces of Newtonian mechanics.
I would prefer to address these gaps with you individually.
Review chapters 1-4 and ask me and each other questions. We have plenty of new things to learn.

0. Review of Newtonian Mechanics

$$F = ma = m\ddot{x}$$

frequently in this course

Generally

$$\dot{x} = \frac{dx}{dt} = v \quad , \quad \ddot{x} = \frac{d^2x}{dt^2} = a$$

$$\overset{\overbrace{\cdots\cdots}^{n \text{ times}}}{x} = \frac{d^n x}{dt^n}$$

That's our whole review of

I. Why begin with oscillations?

Our world is permeated by oscillations, from the yielding sway of tree branches to the bumpy ride of the Bard Shuttle. The mathematics of oscillations runs even deeper than the mechanical examples that come immediately to mind.

In Summary:

- Physical examples surround us.

- Mathematical machinery is invaluable throughout physics: from Electronics to Quantum Field Theory.

We can see a force as arising from a "desire" to minimize potential energy

Mathematically the force is captured by $F = -\, dU/dx$

Why are (1D) oscillations so generic?

Physical systems are often well described by a potential energy:

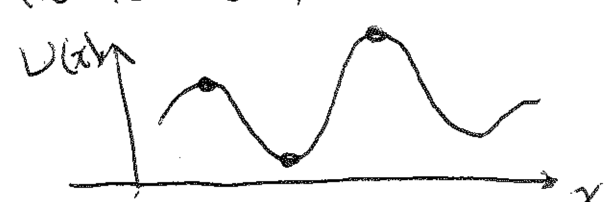
$$U(x)$$

This potential is closely related conceptually to forces because...

Equilibrium is when there is no net force or when

$$F = 0 = \frac{dU}{dx}.$$

This is also the condition for an extremum (max, min or inflection) of the potential



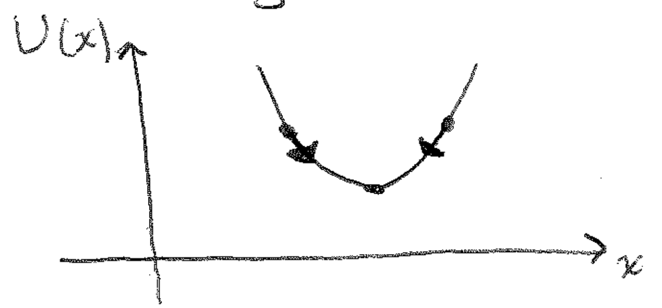
A maximum of the potential energy is an <u>unstable</u> equilibrium. Find 'em by

$$\frac{d^2U}{dx^2} < 0 \quad \left(\text{and } \frac{dU}{dx} = 0 \right)$$

A minimum is a <u>stable</u> equilibrium, Find 'em by

$$\frac{d^2U}{dx^2} > 0$$

a restoring force.



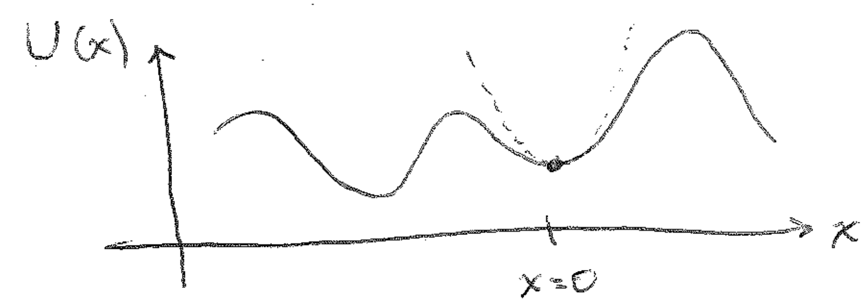
We can say more, Taylor expand the potential near an equilibrium at $x=0$

If

$$\frac{d^2U}{dx^2} = 0$$

the second derivative test fails.

Oscillations arise around stable equilibria! A small displacement (in either direction) pushes the system back towards the equilibrium position — there's



$$U(x) = U(0) + \cancel{U'(0)}^{\,0 \text{ at equilib.}}(x) + \frac{1}{2}U''(0)x^2 + \cdots$$

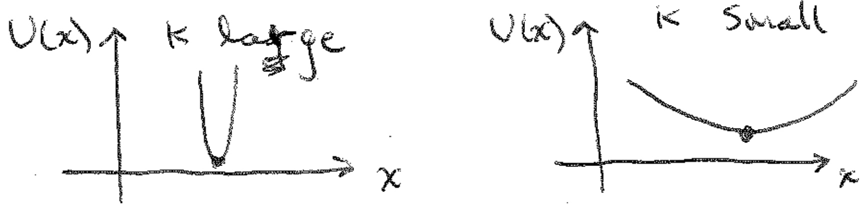
If we shift $U(0)$ to zero we have

$$U(x) = \frac{1}{2}U''(0)\,x^2 + \cdots$$

Compare this to the potential energy of a spring

$$U(x) = \frac{1}{2} k x^2$$

What does the spring constant tell us geometrically? The width of the potential



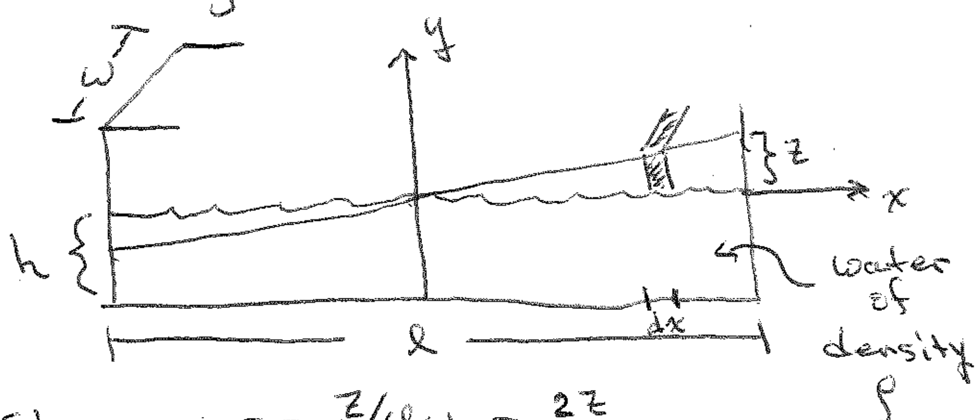
We've shown that

$$k = U''(x_{equilib.})$$

for positive $U''(x_{equilib.})$.

No matter what shape your potential has it looks like a harmonic oscillator near its minima (as long as $U'' > 0$).

Example: Consider water sloshing in a large tub.



Slope $s = z/(\ell/2) = \frac{2z}{\ell}$

and so, $y = \frac{2z}{\ell} x$

$$dm = \rho w y \, dx = \rho w \frac{2z}{\ell} x \, dx$$

The center of mass is at a height $y/2$ for this slab, so

$$U(z) = \int_{-\ell/2}^{\ell/2} \rho w y \cdot \frac{y}{2} g \, dx$$

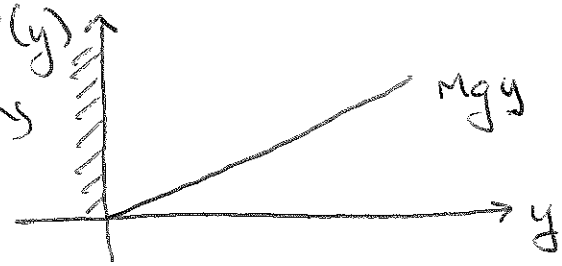
$$= \underbrace{\int_{-\ell/2}^{\ell/2} x^2}_{= \frac{1}{3}\left(\left(\frac{\ell}{2}\right)^3 - \left(-\frac{\ell}{2}\right)^3\right) = \frac{1}{12}\ell^3} \left(\frac{1}{2} \rho w \left(\frac{2z}{\ell}\right)^2 g \right) dx$$

$$= \frac{1}{6} \rho w \ell g z^2 = \frac{1}{2} k z^2 !$$

I hope to have convinced you that oscillation are everywhere.

Now, for a counter example: the bouncy ball

below the floor potential is ∞



Can't do Taylor expansion because it's not differentiable.

Ordinary diff. eqn. (ODE): only ordinary derivatives

Partial diff. eqn. (PDE): partial derivatives

Order of diff. eqn.: The highest deriv.

Most important method of solution is the "standard guess"

$$x(t) = e^{rt}.$$

Change of perspective in this course

$$F = ma = -kx$$

$$\downarrow$$

$$m\frac{d^2x}{dt^2} = -kx$$

$$\downarrow$$

$$m\ddot{x} = -kx$$

We will learn many techniques for writing down eqn.s of motion <u>but also</u> for solving them.