Mechanics
Day 16

Outline

I Last time

II Orbital Transfer

III Noninertial Frames

IV Great Example: Tides

I Summary of Kepler problem

Orbit Eqn:

$$r(\phi) = \frac{c}{1+\epsilon\cos\phi}$$

w/ $c = \frac{\ell^2}{\gamma\mu}$, $\gamma = Gm_1m_2$

$\mu = \frac{m_1m_2}{M}$

Energy/eccentricity

Eccen.	Energy	Orbit
$\epsilon=0$	$E=-\frac{\gamma^2\mu}{2\ell^2}<0$	circle
$0<\epsilon<1$	$E<0$	Ellipse
$\epsilon=1$	$E=0$	parabola
$\epsilon>1$	$E>0$	hyperbola

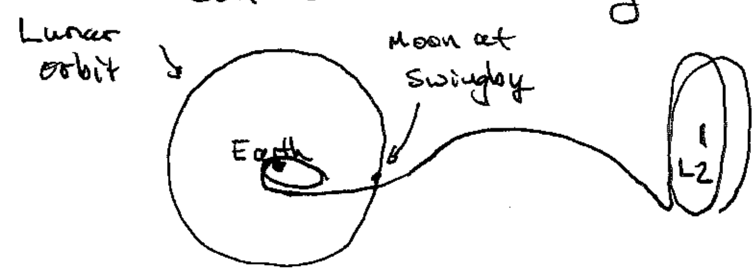
$$E = \frac{\gamma^2\mu}{2\ell^2}(\epsilon^2-1) = \frac{\gamma}{2c}(\epsilon^2-1)$$

• For the orbit transfer

$$r(\phi) = \frac{c}{1+\epsilon\cos(\phi-\delta)}$$

A brief <u>thrust</u> results in a known change in velocity from which we find $E_1 \to E_2$ and $\ell_1 \to \ell_2$.

These can be wonderfully complex:



However, We'll begin with some interesting and simpler examples.

II If you are interested in the general case, it is useful to outline the steps for figuring it out!

From l_1, l_2 we find

$$c_1 = l_1^2/\gamma\mu \quad c_2 = l_2^2/\gamma\mu$$

and from $E = \gamma/2c\,(\epsilon^2 - 1)$ we find ϵ_1 and ϵ_2. Finally, assuming the thrust occured at precisely ϕ_0 (an approximation) we equate the two orbit equations at this point to find δ_2 from δ_1,

$$\frac{c_1}{1+\epsilon_1\cos(\phi_0-\delta_1)} = \frac{c_2}{1+\epsilon_2\cos(\phi_0-\delta_2)}$$

Let λ, the "thrust factor", be s.t.

$$v_2 = \lambda v_1$$

($\lambda > 1$ speed up ; $\lambda < 1$ slow down).

At perigee $v = r\dot{\phi}$ and $l = \mu r v$ then, assuming the thrust doesn't significantly change the satellite mass,

$$l_2 = \lambda l_1$$

and

$$c_2 = \lambda^2 c_1$$

That's the whole story but there's
lots of algebra and instead all the problems tend to focus on simpler versions.

<u>Tangential thrust at perigee</u>:

Choose thrust s.t. $\phi = 0 \Rightarrow \phi_0 = 0$

$\delta_1 = 0$, $\delta_2 = 0$:

$$c_1/(1+\epsilon_1) = \frac{c_2}{(1+\epsilon_2)}$$

$$\Rightarrow \frac{c_1}{1+\epsilon_1} = \frac{\lambda^2 c_1}{(1+\epsilon_2)}$$

$$\Rightarrow \epsilon_2 = \lambda^2(1+\epsilon_1) - 1$$
$$= \lambda^2\epsilon_1 + (\lambda^2 - 1)$$

$\lambda > 1 \Rightarrow \epsilon_2 > \epsilon_1$, orbit is more eccentric until escape.

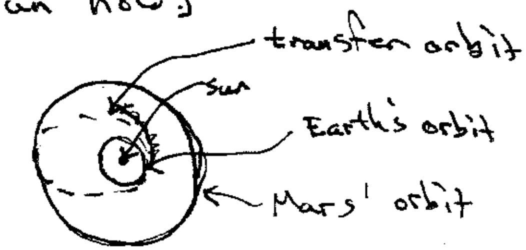
$\lambda < 1 \Rightarrow \epsilon_2 < \epsilon_1$ less eccentric $\rightarrow$ circular $\rightarrow$ perigee and apogee switch!

<u>Fun example</u>: Most efficient way to get to Mars: "the Hohmann transfer".

Both the Earth and Mars have roughly circular orbits:

	<u>Eccentricity</u>	<u>Orbit radius</u>
Earth	$\epsilon_e = 0.0167$	$R_e = 1.5 \times 10^{11}$ m
Mars	$\epsilon_M = 0.0933$	$R_M = 2.3 \times 10^{11}$ m

The Hohmann transfer involves two thrusts, the first takes you from the circular (earth) orbit to an elliptical orbit that intersects the desired orbit (Mars in our case) and the second which brings you onto the target orbit. (Note: I'm speaking about orbits about the sun now)



Take $\epsilon_e \approx \epsilon_M \approx 0$. Denote the transfer orbit with t subscripts. Then

$$c_1 = c_e = R_e$$

and $c_t = \lambda^2 R_e \quad \epsilon_t = \lambda^2(0) + (\lambda^2 - 1)$
$= (\lambda^2 - 1)$

Now R_M must be aphelion of transfer so,
aphelion of transfer orbit

$$R_M = \frac{c_t}{1-\epsilon_t} = \frac{\lambda^2 R_e}{1-(\lambda^2-1)} = \frac{\lambda^2 R_e}{2-\lambda^2}$$

$$\Rightarrow 2R_M - \lambda^2 R_M = \lambda^2 R_e \Rightarrow \lambda^2(R_e + R_M) = 2R_M$$

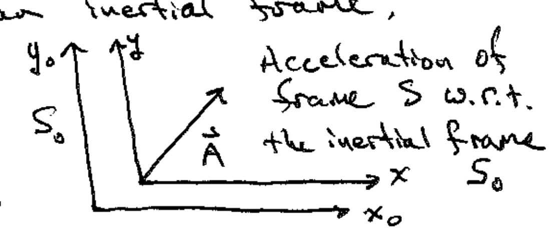
$$\Rightarrow \lambda = \sqrt{\frac{2R_M}{R_e+R_M}} = \sqrt{\frac{4.6}{3.8}} = 1.10$$

Need a 10% increase in speed!

I leave to you to find λ_2, the thrust factor needed to go into Mars' orbit.

III. • Newton's laws only hold in inertial frames
• We are careful to write down the Lagrangian in an inertial frame.

For the next few meetings we explore non-inertial frames



Consider the position of a particle. In the frame S_0 this is measured to be $\vec{r}_0$ and it obeys Newton's 2nd law

$$m\ddot{\vec{r}}_0 = \vec{F}$$

In S they measure $\vec{r}$ and

$$\dot{\vec{r}}_0 = \dot{\vec{r}} + \vec{V}$$

particle's vel. w.r.t. ground = particle's vel. w.r.t. moving frame + vel. of moving frame w.r.t. ground

This then implies

$$\ddot{\vec{r}}_0 = \ddot{\vec{r}} + \vec{A} \quad \text{or} \quad \ddot{\vec{r}} = \ddot{\vec{r}}_0 - \vec{A}$$

so that

$$m\ddot{\vec{r}} = m\ddot{\vec{r}}_0 - m\vec{A}$$
$$= \vec{F} - m\vec{A}$$

This looks a lot like Newton's 2nd law. In fact, it is if we agree to introduce an "inertial force"

$$\vec{F}_{\text{inertial}} = -m\vec{A}$$