# Homework 2

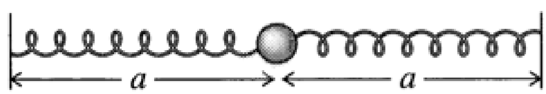
Due Thursday, September 13th at 5pm

Finish Chapter 5 and begin Chapter 6 of Taylor's Classical Mechanics.

1. A large Foucault pendulum such as hangs in many science museums can swing for many hours before it damps out. Taking the decay time to be about 8 hours and the length to be 30 meters, find the quality factor $Q$.

2. Consider the two-dimensional anisotropic oscillator with motion given by Taylor's Equation (5.23). (**a**) Prove that if the ratio of frequencies is rational (that is, $\omega_x/\omega_y = p/q$ where $p$ and $q$ are integers) then the motion is periodic. What is the period? (**b**) Prove that if the same ratio is irrational, the motion never repeats itself.

3. The mass shown in the Figure is resting on a frictionless horizontal table.




   Each of the two identical springs has force constant $k$ and unstretched length $l_o$. At equilibrium the mass rests at the origin, and the distances a are not necessarily equal to $l_o$. (That is, the springs may already be stretched or compressed.) Show that when the mass moves to a position $(x, y)$, with $x$ and $y$ small, the potential energy has the form for an anisotropic oscillator:

$$U = \frac{1}{2}(k_x x^2 + k_y y^2) \tag{1}$$

   Show that if $a < l_o$ the equilibrium at the origin is unstable and explain why.

4. Consider the mass attached to four identical springs, as shown in Taylor's Figure 5.7(b). Each spring has force constant $k$ and unstretched length $l_o$, and the length of each spring when the mass is at its equilibrium at the origin is a (not necessarily the same as $l_o$ ). When the mass is displaced a small distance to the point $(x, y)$, show that its potential energy has the form $\frac{1}{2}k'r^2$ appropriate to an isotropic harmonic oscillator. What is the constant $k'$ in terms of $k$? Give an expression for the corresponding force. [Hint: Be sure to take advantage of all the work you did in the last problem so you don't have to do as many calculations for this one.]

5. The position $x(t)$ of an overdamped oscillator is given by Taylor's Eq. (5.40). (a) Find the constants $C_1$ and $C_2$ in terms of the initial position $x_o$ and velocity $v_o$. (b) Sketch the behavior of $x(t)$ for the two cases that $v_o = 0$ and that $x_o = 0$. (c) To illustrate again how mathematics is sometimes cleverer than one might expect (and to check your answer), show that if you let $\beta \to 0$, your solution for $x(t)$ in part (a) approaches the correct solution for undamped motion.

6. Particular Solutions. This problem justifies my claim in lecture that you should find particular solutions by hook or by crook, that is, by any method you find convenient. On Friday we showed that the general solution of an inhomogeneous differential equation was the sum of a solution to the homogeneous equation $x_h$ and a particular solution $x_p$ to the inhomogeneous

equation. Show that it doesn't matter what particular solution you use (!). [Hints: Assume you've found two particular solutions $x_{p1}$ and $x_{p2}$, what is special about the difference $(x_{p1} - x_{p2})$ of these two solutions? Try acting $D$ on this difference. Argue that because of this special property you can switch from one particular solution to the other by changing the constants in the homogeneous solution $x_h$.]