Homework 3

Due Friday, September 21st at 5pm

Finish Chapter 6 and begin Chapter 7 of Taylor's Classical Mechanics.

1. When a car drives along a "washboard" road, the regular bumps cause the wheels to oscillate on the springs. (What actually oscillates is each axle assembly, comprising the axle and its two wheels.) Find the speed of my car at which this oscillation resonates, given the following information: (a) When four 80-kg men climb into my car, the body sinks by two centimeters. Use this to estimate the spring constant k of each of the four springs. (b) If an axle assembly (axle plus two wheels) has total mass 50 kg, what is the natural frequency of the assembly oscillating on its two springs? (c) If the bumps on a road are 80 cm apart, at about what speed would these oscillations go into resonance?

2. Another interpretation of the Q of a resonance comes from the following: Consider the motion of a driven damped oscillator after any transients have died out, and suppose that it is being driven close to resonance, so you can set $\omega = \omega_o$. (**a**) Show that the oscillator's total energy (kinetic plus potential) is $E = \frac{1}{2}m\omega^2 A^2$. (**b**) Show that the energy ΔE_{dis} dissipated during one cycle by the damping force F_{dmp} is $2\pi m\beta\omega A^2$. (Remember that the rate at which a force does work is Fv.) (c) Hence show that Q is 2π times the ratio $E/\Delta E_{\text{dis}}$

3. Consider a damped oscillator, with natural frequency ω_o and damping constant β both fixed, that is driven by a force $F(t) = F_o \cos(\omega t)$. (a) Find the rate $P(t)$ at which $F(t)$ does work and show that the average rate $\langle P \rangle$ over any number of complete cycles is $m\beta\omega^2 A^2$. (b) Verify that this is the same as the average rate at which energy is lost to the resistive force. (c) Show that as ω is varied $\langle P \rangle$ is maximum when $\omega = \omega_o$; that is, the resonance of the power occurs at $\omega = \omega_o$ (exactly).

4. Taylor 6.3, p231

5. Taylor 6.5, p231

6. Taylor 6.1 & 6.16, pp 230 & 233

Optional: The following problem from Taylor is not part of this week's assignment. However, it will be assigned as part of a future week's homework. So, if you like, you can do it this week.

- Taylor 6.25, p234

The reason I include it here is that it has a nice connection with the oscillations material you've been thinking about. A regular pendulum makes a very good clock (this was Galileo's astonishing insight) but it doesn't make a perfect one. (Why not? Hint: There are multiple reasons.) But by thinking about the results of Taylor 6.25 Huygens' concocted a brilliant pendulum that is theoretically a perfect clock! Can you think of a way to construct such a pendulum? [Hint: It's a bit like your homework problem with the pendulum wrapped around the cylinder. You want to change how the pendulum swings.] Unfotunately, Huygens' idea isn't practical; it turns out that there is too much friction and it ends up being a worse clock in practice then the good old pendulum bob.