Classical Mechanics

Day 1

Today:

0. Syllabus

I. Review Newtonian Mechanics

II. Why are simple harmonic oscillations everywhere?

III The Standard Guess

I. Newtonian Mechanics

$$F = ma \underset{\uparrow \text{ notation}}{=} m\ddot{x}$$

Generally,

$$\dot{x} \equiv \frac{dx}{dt} = v, \qquad \ddot{x} \equiv \frac{d^2x}{dt^2} = a$$

$$\overbrace{\dot{\ddot{\ddot{x}}}}^{n \text{ times}} \equiv \frac{d^n x}{dt^n}$$

That's our whole review.

Why? All of you are missing different pieces of Newtonian Mechanics. I would prefer to address these gaps with you individually. Please review Ch.s 1-4 and ask me and each other questions. We have plenty of new things to learn.

I. Why begin with oscillations?

They are everywhere, from the yielding sway of tree branches to the bumpy ride of the Bard shuttle. The mathematics of oscillations runs even deeper than the mechanical examples. They appear throughout physics from Electronics to Quantum Field Theory.

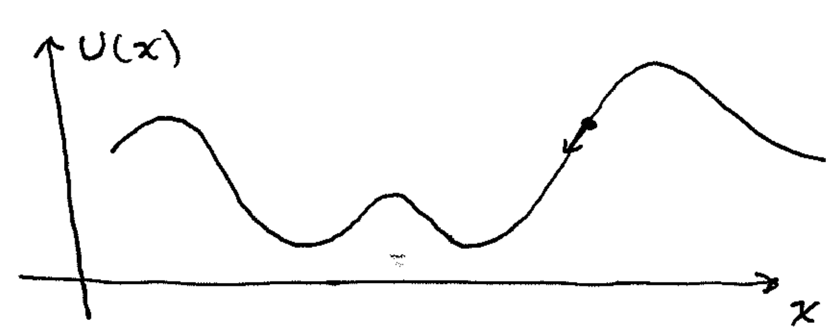
<u>Why</u> are (1D) oscillations so generic?

Physical systems are often well described by a potential energy (PE):

$$U(x).$$

This potential is closely related to forces because...

... we can see a force as arising from a "desire" to minimize PE:



Mathematically the force is captured by

$$F = -dU/dx.$$

Equilibrium is when there is no net force, or when

$$F = 0 = -dU/dx.$$

But, this is also the condition for an extremum = (max, min or inflection) of the potential

A maximum of the PE is an <u>unstable</u> equilibrium. Find 'em by

$$\frac{d^2U}{dx^2} < 0 \quad \left(\text{and } dU/dx = 0 \right).$$

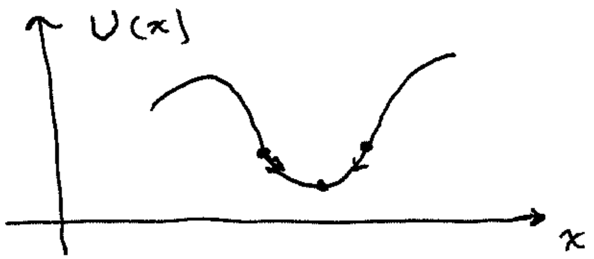
A min is a <u>stable</u> equilib.

Find 'em by

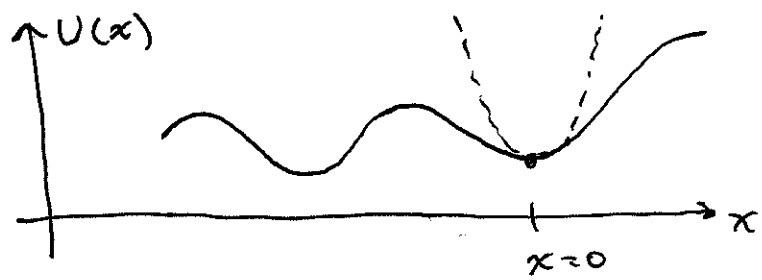
$$\frac{d^2U}{dx^2} > 0$$

<u>If</u> $d^2U/dx^2 = 0$ 2^{nd} derivative test fails.

Oscillations arise around stable equilibria! A small displacement (in either direction) results in a force back towards the equilib. position — there's a restoring force:



We can say more,
Taylor expand the potential near an equilib at $x = 0$



$$U(x) = U(0) + \underbrace{\cancel{U'(0)}}_{0 \text{ at equilib position}} x + \frac{1}{2} U''(0) x^2 + \cdots$$

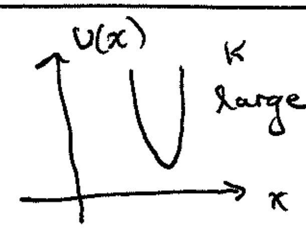
If we shift $U(0)$ to zero we have

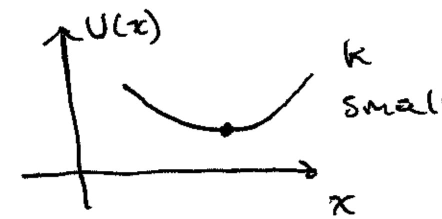
$$U(x) = \frac{1}{2} U''(0) x^2 + \cdots$$

Compare this to the PE of a spring:

$$\boxed{U(x) = \frac{1}{2} k x^2}$$

What does the spring constant tell us geometrically?
The "width" or curvature of the potential (see pictures)



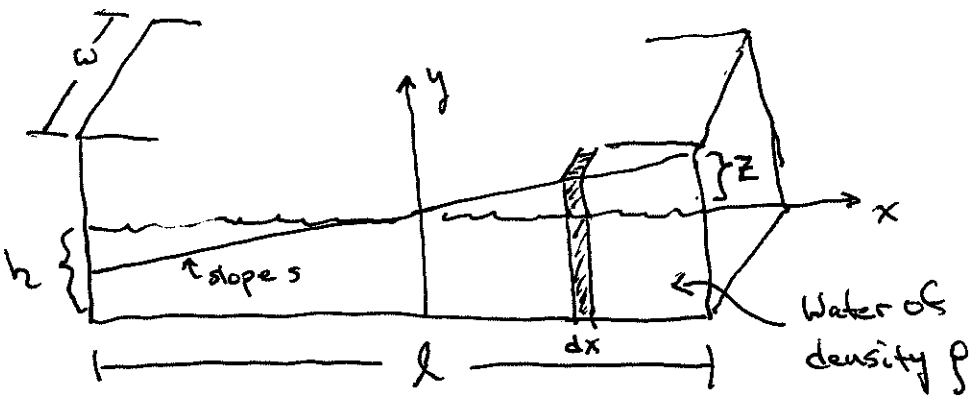


We've shown that

$$\boxed{k = U''(x_{equilib}),}$$

for positive $U''(x_{equilib})$.
No matter its shape a potential looks like an harmonic oscillator near its minima (as long as $U'' > 0$).

Example: Consider water sloshing in a fish tank:



Slope $s = z/(l/2) = \frac{2z}{l}$

and so, $y = \frac{2z}{l}x$

$$dm = \rho w y\, dx = \rho w \frac{2z}{l} x\, dx.$$

The center of mass of the slab is at a height $y/2$, so

$$U(z) = \int_{-l/2}^{l/2} \rho w y \cdot \frac{y}{2} g\, dx$$

$$= \int_{-l/2}^{l/2} \left(\frac{1}{2}\rho w \left(\frac{2z}{l}\right)^2\right) x^2\, dx$$

$$= 2\rho w \frac{z^2}{l^2} \cdot \frac{1}{3}\left[\left(\frac{l}{2}\right)^3 - \left(-\frac{l}{2}\right)^3\right]$$

so,

$$U(z) = \frac{1}{6}\rho w l g\, z^2 \equiv \frac{1}{2} k z^2 \; !$$

It is harmonic! The spring constant is

$$\boxed{k = \frac{1}{3}\rho w l g}$$

Note carefully that the harmonic potential has a $1/2$ in it and so k must be adjusted accordingly.