E & M
Day 1

<u>Today</u>

I Review Syllabus

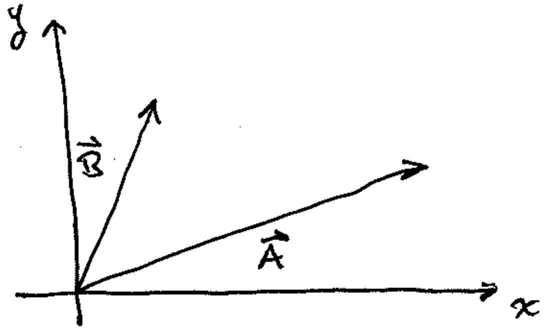
II Hal Guest Lecture:
Vector Analysis

III Bruno Guest Lecture:
Differential Calculus

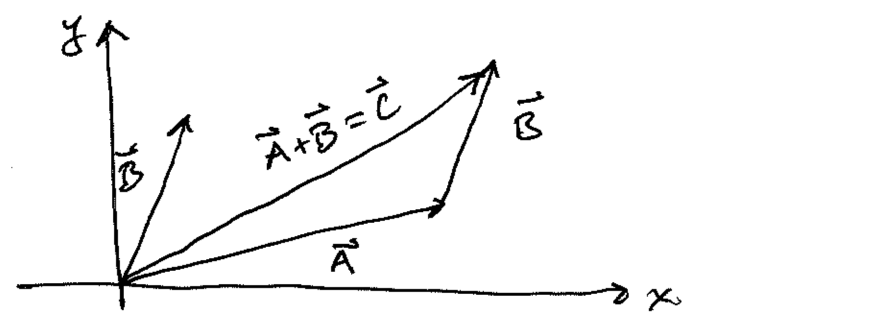
I We reviewed the Syllabus and set three office hours:
Th 4-5pm, F 3-5pm.

II Main goal: review the notation we will use throughout the semester.
I'll do this by reviewing the graphical addition and subtraction of vectors. Of course, this can also be done algebraically using vector components.

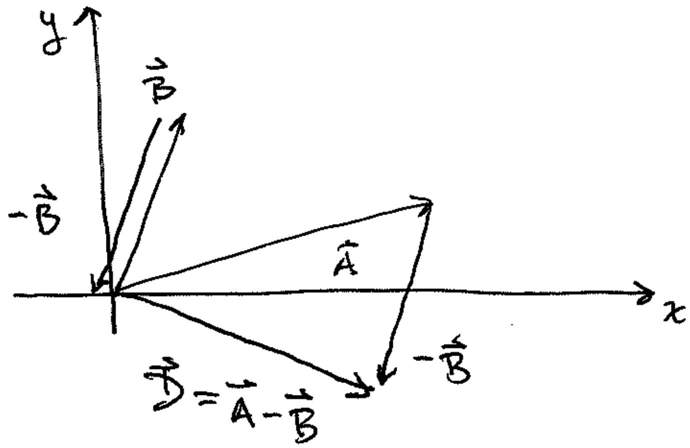
Consider $\vec{A}$ and $\vec{B}$ 2D vectors



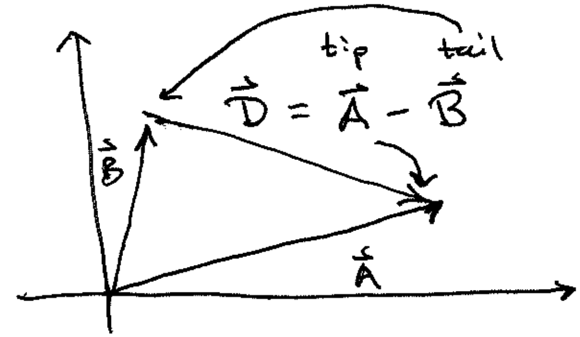
We obtain their sum $\vec{C} = \vec{A} + \vec{B}$ by dragging either vector along the other and then drawing an arrow from the base of the 1st to the tip of the 2nd



The subtraction of vectors works similarly $\vec{D} = \vec{A} - \vec{B} = \vec{A} + (-\vec{B})$

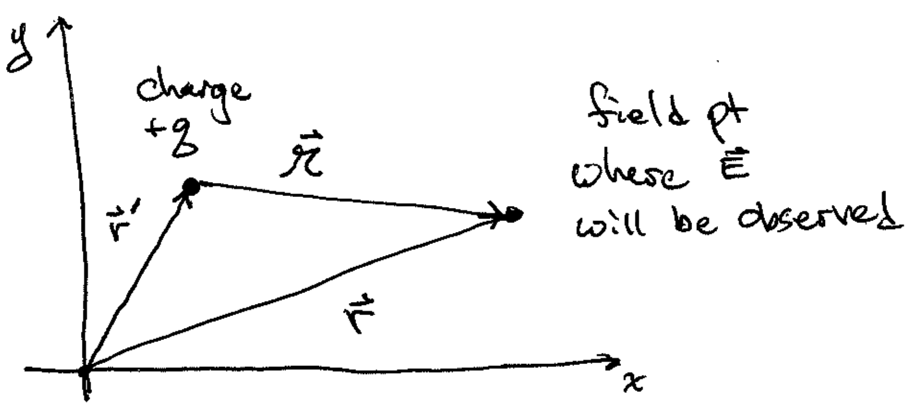


There's a shortcut, which is captured by the mnemonic "tip minus tail"



Notice that both ways of obtaining $\vec{D}$ are the same, since the two vectors are parallel.

<u>Notation in E&M</u>: This result will be very useful to us:



Coordinate System

The electric field will always point directly away from a positive charge towards where it is being observed, thus the vector

$$\vec{\mathscr{r}} = \vec{r} - \vec{r}'$$

is very useful.

What is r? $r = |\vec{r}| \equiv \sqrt{\vec{r} \cdot \vec{r}}$

Or in coordinates

$$\vec{r} = x\hat{x} + y\hat{y} + z\hat{z},$$

so,

$$\vec{r}\cdot\vec{r} = x^2 + y^2 + z^2$$

and

$$r = \sqrt{x^2 + y^2 + z^2}.$$

This is the vectorial way to capture the pythagorean theorem.

Similarly $\vec{r}\,' = x'\hat{x} + y'\hat{y} + z'\hat{z}$.

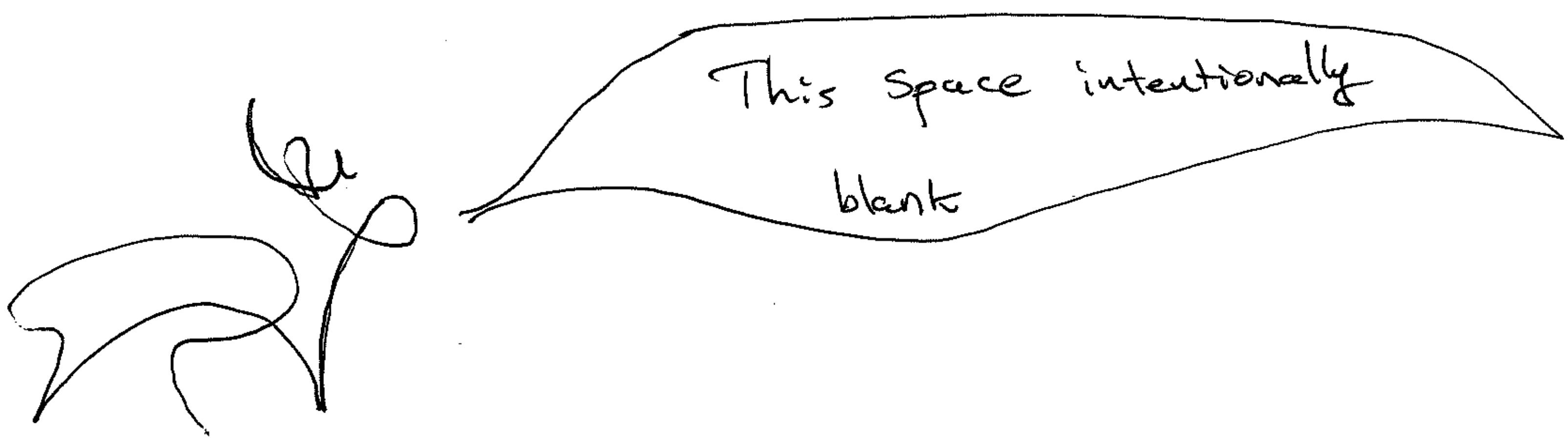
Then

$$\vec{\mathscr{r}} = (x-x')\hat{x} + (y-y')\hat{y} + (z-z')\hat{z}$$

and

$$\mathscr{r} = \sqrt{(x-x')^2 + (y-y')^2 + (z-z')^2}.$$

This is a result that will be very useful to you.



Bruno's guest lecture on Differential Calculus

Given $f(x)$,

$$df = \left(\frac{df}{dx}\right) dx.$$

Let's generalize to

$T = T(x, y, z)$, e.g. the temperature in the room.

Define the gradient by

$$\vec{\nabla} T\Big|_p = \left(\frac{\partial T}{\partial x}\hat{x} + \frac{\partial T}{\partial y}\hat{y} + \frac{\partial T}{\partial z}\hat{z}\right)\Bigg|_p$$

We can relate this to

$$dT = \vec{\nabla} T \cdot d\vec{l}$$
$$= |\vec{\nabla} T||d\vec{l}|\cos\theta$$

where $d\vec{l} = dx\,\hat{x} + dy\,\hat{y} + dz\,\hat{z}$.

The gradient points in the direction of greatest increase of the function.

~~To~~ Can see this from

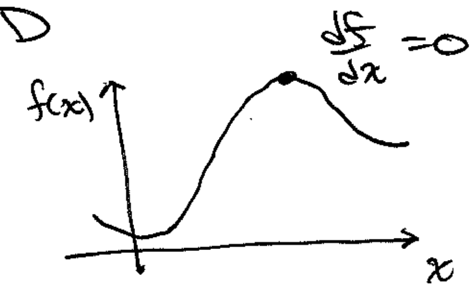
$$dT = |\vec{\nabla} T||d\vec{l}|\cos\theta$$

which is max when $\theta = 0$, i.e. when $d\vec{l}$ is aligned with $\vec{\nabla} T$.

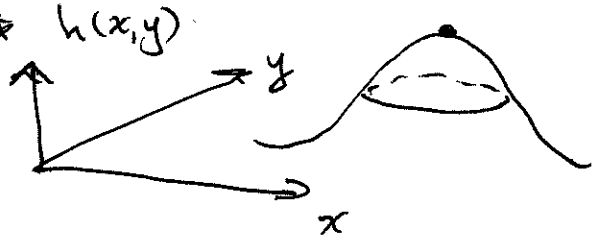
Given a function describing the height of a hill $h = h(x,y)$,

$$h = 10\left(2xy - 3x^2 - 4y^2 - 18x + 28y + 12\right)$$

In 1D



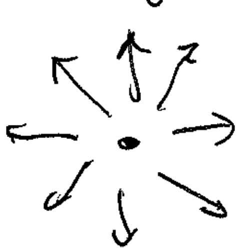
In 2D the top of the hill is $h(x,y)$ at



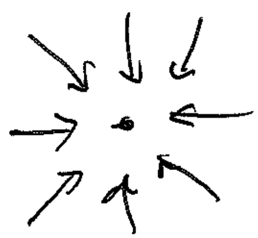
$$\vec{\nabla} h(x,y) = 0 \quad \begin{cases} \partial h/\partial x = 0 \\ \text{and} \\ \partial h/\partial y = 0. \end{cases}$$

The divergence gives the amount of flow out of a pt (where it's evaluated).

$\vec{v}$

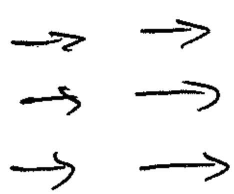


$\vec{\nabla} \cdot \vec{v} > 0$



$\vec{\nabla} \cdot \vec{v} < 0$

$\vec{v}$



$\vec{\nabla} \cdot \vec{v} = 0$

The Divergence

$$\vec{\nabla} \cdot \vec{v} = \left(\frac{\partial}{\partial x}\hat{x} + \frac{\partial}{\partial y}\hat{y} + \frac{\partial}{\partial z}\hat{z} \right) \cdot \left(v_x \hat{x} + v_y \hat{y} + v_z \hat{z} \right)$$

$$= \frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z}$$

Note that the component v_x of the vector field $\vec{v}$ is a scalar field $v_x = v_x(x,y,z)$.

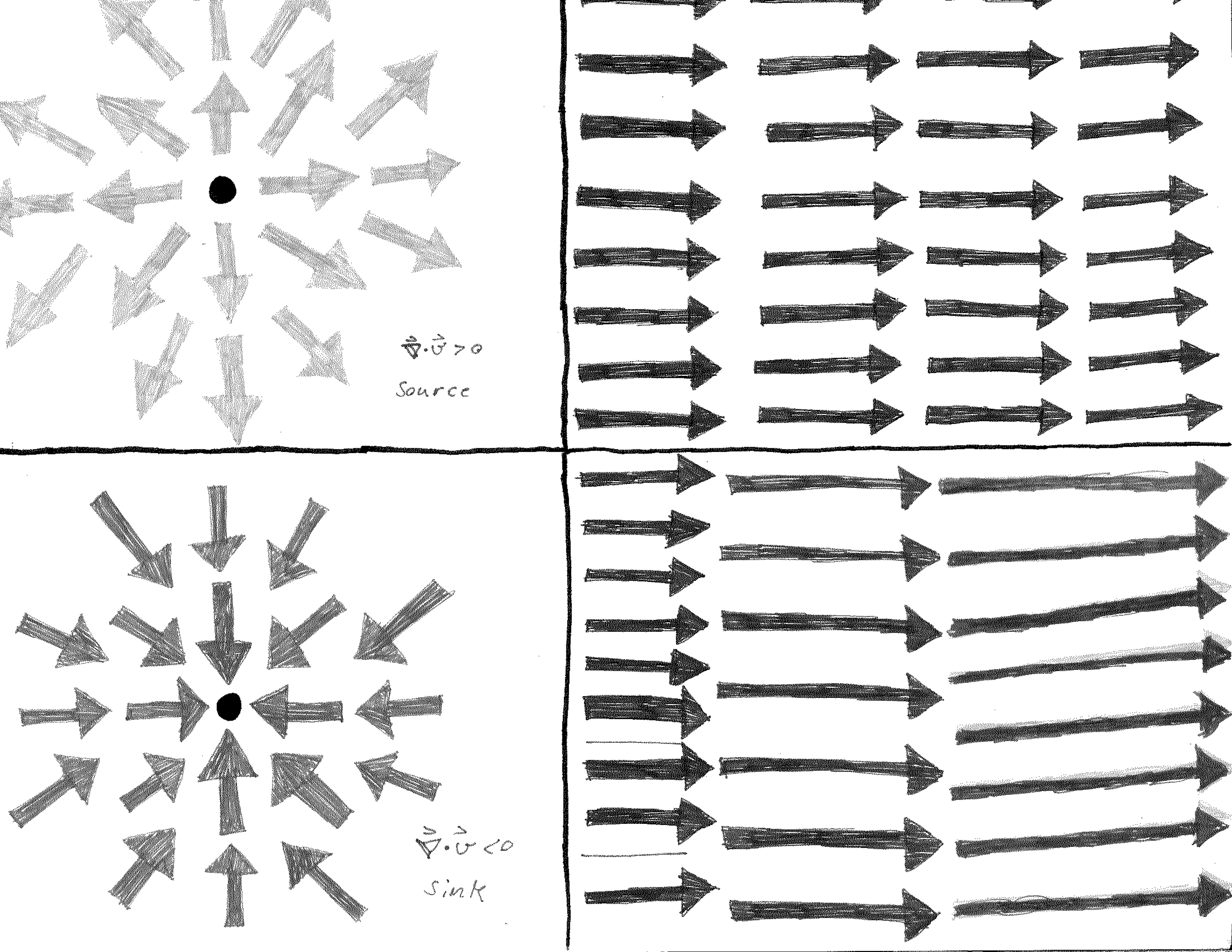
Ex. $\vec{v} = x^2 \hat{x} + x z^2 \hat{y} - 2xz\hat{z}$

$$\vec{\nabla} \cdot \vec{v} = 2x + 0 - 2x = 0.$$

The Curl

$$\vec{\nabla} \times \vec{v} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ v_x & v_y & v_z \end{vmatrix}$$

The curl gives the amount the $\vec{v}$ field swirls around pt. P.


$\vec{\nabla}\cdot\vec{v} > 0$
Source
$\vec{\nabla}\cdot\vec{v} < 0$
Sink

Product Rules.

2 ways to Construct a Scalar as a Product

$\underline{fg}$ $\qquad$ $\underline{\vec{A}\cdot\vec{B}}$

2 ways to make a vector

$\underline{f\vec{A}}$ $\qquad$ $\underline{\vec{A}\times\vec{B}}$

(i) $\vec{\nabla}(fg) = f\vec{\nabla}g + g\vec{\nabla}f$

(ii) $\vec{\nabla}(\vec{A}\cdot\vec{B}) = \vec{A}\times(\vec{\nabla}\times\vec{B}) + \vec{B}\times(\vec{\nabla}\times\vec{A}) + (\vec{A}\cdot\vec{\nabla})\vec{B}$

(iii) $\vec{\nabla}\cdot(f\vec{A}) = f(\vec{\nabla}\cdot\vec{A}) + \vec{A}\cdot(\vec{\nabla}f)$

(iv) $\vec{\nabla}\times(\vec{A}\times\vec{B}) = \vec{B}\cdot(\vec{\nabla}\times\vec{A}) - \vec{A}\cdot(\vec{\nabla}\times\vec{B})$

(v) $\vec{\nabla}\times(f\vec{A}) = f(\vec{\nabla}\times\vec{A}) - \vec{A}\times(\vec{\nabla}f)$

(vi) $\vec{\nabla}\times(\vec{A}\times\vec{B}) = (\vec{B}\cdot\vec{\nabla})\vec{A} - (\vec{A}\cdot\vec{\nabla})\vec{B} + \vec{A}(\vec{\nabla}\cdot\vec{B}) - \vec{B}(\vec{\nabla}\cdot\vec{A})$