Homework #3

Due September 19, 2014

Reading: Chapter 3 of Reif.

1. Reif 1.13, p42.
2. Reif 1.21, p44.
3. Reif 2.10, p86.
4. Reif 2.11, p86.
5. Schroeder 1.16, p8:

The exponential atmosphere.

(a) Consider a horizontal slab of air whose thickness (height) is dz. If this slab is at rest, the pressure holding it up from below must balance both the pressure from above and the weight of the slab. Use this fact to find an expression for $d\bar{p}/dz$, the variation of pressure with altitude, in terms of the density of air.

(b) Use the ideal gas law to write the density of air in terms of pressure, temperature, and the average mass m of the air molecules. (You can calculate m using the fact that a mole of dry air, which is a mixture of N_2 (78% by volume), O_2 (21%), and argon (1%).) Show, then, that the pressure obeys the differential equation

$$\frac{d\bar{p}}{dz} = -\frac{mg}{kT}\bar{p}$$

called the **barometric equation**.

(c) Assuming that the temperature of the atmosphere is independent of height (not a great assumption but not terrible either), solve the barometric equa- tion to obtain the pressure as a function of height: $\bar{p}(z) = \bar{p}(0)e^{-mgz/kT}$. Show also that the density obeys a similar equation.

(d) Estimate the pressure, in atmospheres, at the following locations: Ogden, Utah (4700 ft or 1430 m above sea level); Leadville, Colorado (10,150 ft, 3090 m); Mt. Whitney, California (14,500 ft, 4420 m); Mt. Everest, Nepal! Tibet (29,000 ft, 8840 m). (Assume that the pressure at sea level is 1 atm.)

Note: For this problem I am assuming one piece of background, the ideal gas law, that we won't derive from the microscopic picture until the end of this week or beginning of next week. You will probably have seen this law before; it relates the (average) pressure $\bar{p}$, volume V, number of molecules in that volume N, and temperature T by

$$\bar{p}V = NkT,$$

where k is the Boltzmann constant. A second form re-expresses the number of molecules in terms of the number of moles ν and ideal gas constant $R = 8.315 J/mol \cdot K$ as $Nk = \nu R$, so that the ideal gas law becomes,

$$\bar{p}V = \nu RT.$$