Homework #4

Due September 19, 2014

Reading: Chapter 3 of Reif.

1. Reif 3.3, p127.
2. Reif 3.4, p127.
3. Reif 3.5, p127.
4. Reif 3.6, p127.
5. Schroeder 1.38, p26:

Two identical bubbles of gas form at the bottom of a lake, then rise to the surface. Because the pressure is much lower at the surface than at the bottom, both bubbles expand as they rise. However, bubble A rises very quickly, so that no heat is exchanged between it and the water. Meanwhile, bubble B rises slowly (impeded by a tangle of seaweed), so that it always remains in thermal equilibrium with the water (which has the same temperature everywhere). Which of the two bubbles is larger by the time they reach the surface? Explain your reasoning fully.

6. Schroeder 1.40, p27. In Problem 1.16 you calculated the pressure of earth's atmosphere as a function of altitude, assuming constant temperature. Ordinarily, however, the temperature of the bottommost 10- 15 km of the atmosphere (called the troposphere) decreases with increasing altitude, due to heating from the ground (which is warmed by sunlight). If the temperature gradient $|dT/dz|$ exceeds a certain critical value, convection will occur: Warm, low-density air will rise, while cool, high-density air sinks. The decrease of pressure with altitude causes a rising air mass to expand adiabatically and thus to cool. The condition for convection to occur is that the rising air mass must remain warmer than the surrounding air despite this adiabatic cooling. (a) Show that when an ideal gas expands adiabatically, the temperature and pressure are related by the differential equation

$$\frac{dT}{d\bar{p}} = \frac{2}{\hat{f}+2}\frac{T}{\bar{p}},$$

here $\hat{f}$ is the number of degrees of freedom per molecule.

(b) Assume that dT/dz is just at the critical value for convection to begin, so that the vertical forces on a convecting air mass are always approximately in balance. Use the result of Problem 1.16(b) to find a formula for dT/dz in this case. The result should be a constant, independent of temperature and pressure, which evaluates to approximately $-10°C/km$. This fundamental meteorological quantity is known as the dry adiabatic lapse rate.