# Thermal Physics

## Meeting XII

I Last time

II Review of $2^{nd}$ law logic.

III Real Heat Engines

IV Refrigerators

• Free expansion

$T_2 = T_1$ ideal gas

$T_2 < T_1$ van der Waals gas

Comes from cons. of energy.

• Throttling

Enthalpy conserved

$H(T_2, p_2) = H(T_1, p_1)$

Led to consideration of

$$\mu \equiv \left(\frac{\partial T}{\partial p}\right)_H$$

the "Joule-Thompson" coeff.

$\mu > 0 \rightsquigarrow$ cooling of gas

• Heat Engines

Perfect engine is impossible

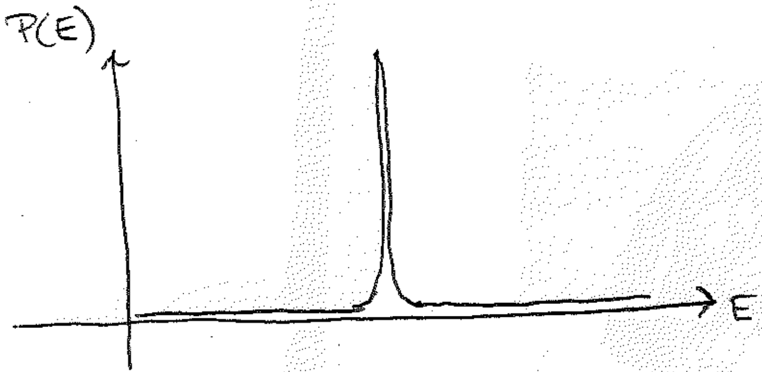
$$\boxed{T_H} \xrightarrow{q} \text{(M)} \xrightarrow{} w$$

perfect if $q = w$.

Not possible!

II Review of $2^{nd}$ law logic

Recall




because of the huge # of d.o.f.

But then when

$\boxed{A, T_i}$ $\boxed{A', T_i'}$

are put into thermal contact 
they are in very low probability state ⇝ the system evolves toward higher probability, which is achieved when

$$\beta_f = \beta_f' \quad \text{or} \quad T_f = T_f'$$

The final probability is maximum and hence can't be less than the initial one!

$$P(E) = C\,\Omega(E)\,\Omega(E')$$

with $E' = E^{(0)} - E$. Then

$$\ln P(E) = \ln C + \ln\Omega + \ln\Omega'$$

and we learn

$$S_f + S_f' \geq S_i + S_i'$$

When does equality hold?

$$\Delta S_{tot} = \Delta S + \Delta S' \geq 0. \quad 2^{nd}\text{ law.}$$

If the ~~to~~ two systems are already in equilibrium then certainly $\Delta S_{tot} = 0$ (already at prob. max.) More generally if they are infinitessimally seperated

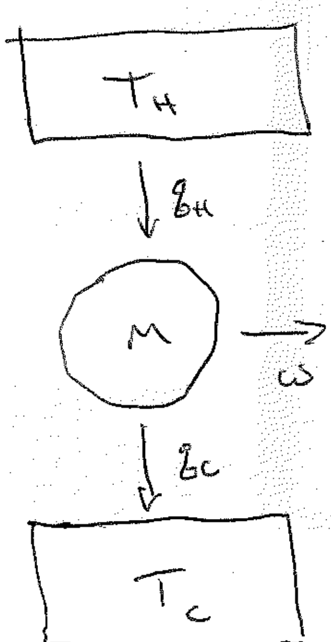
$$dS = \frac{đQ}{T} \overset{\text{adiabatic}}{=} 0 \Rightarrow \Delta S = 0$$

for quasi-static.

Our proof that there was no perfect engine was just the 2nd law. Hence it is a form of the 2nd law

It is impossible to construct a perfect heat engine.

Kelvin's formulation of 2nd law




and now the 2nd law gives

$$\Delta S = \frac{-q_H}{T_H} + \frac{q_c}{T_c} \geq 0$$

Putting these together

$$-\frac{q_H}{T_H} + \frac{q_H - w}{T_c} \geq 0$$

III Instead of trying to extract energy from random motion and turn it purely into work we can do some work and cause some random motion

Conservation of energy is

$$q_H = w + q_c$$

$$\Rightarrow \quad \frac{w}{T_c} \leq q_H \left( \frac{1}{T_c} - \frac{1}{T_H} \right)$$

Introduce "efficiency"

$$\eta \equiv \frac{w}{q_H}$$

then

$$\boxed{\eta = \frac{w}{q_H} \leq 1 - \frac{T_c}{T_H} = \frac{T_H - T_c}{T_H}}$$

For a perfect engine

$$\eta = \frac{w}{q_H} = 1 \quad \text{(not possible)}$$

in reality

$$\eta = \frac{q_H - q_c}{q_H} < 1$$

Best we can do is a quasi-static engine with

$$\eta = \frac{T_H - T_c}{T_H}$$

<u>Ideal gas</u>: $pV = \nu RT$

and $PV^{\gamma} = \text{const.}$ adiabat

Heat along isotherms

$b \to c \quad dE = đQ - dW$

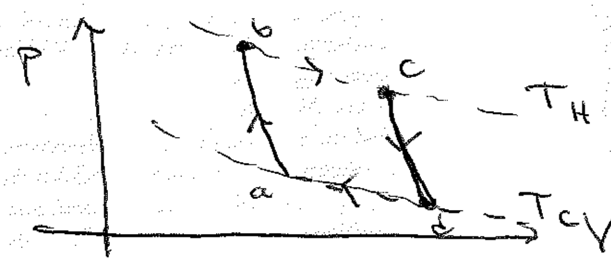
isotherm $\Rightarrow dE = 0$ so

$đQ = dW$

or $q_H = \int_b^c p\,dV = \int_{V_b}^{V_c} \frac{\nu RT_H}{V} dV = \nu RT_H \ln\left(\frac{V_c}{V_b}\right)$

What engine realizes this maximum efficiency?

A Carnot engine; which consists of two adiabats and two isotherms




Similarly,

$$q_c = \nu RT_c \ln\left(\frac{V_d}{V_a}\right)$$

Then

$$\eta = \frac{w}{q_H} = \frac{q_H - q_c}{q_H} = 1 - \frac{\nu RT_c \ln\left(\frac{V_d}{V_a}\right)}{\nu RT_H \ln\left(\frac{V_c}{V_b}\right)}$$

$$= 1 - \frac{T_c}{T_H} \frac{\ln\left(\frac{V_d}{V_a}\right)}{\ln\left(\frac{V_c}{V_b}\right)}$$

How to find Vol ratios?

We did this together. Use the fact that $a \to b$ and $c \to d$ are adiabats while $b \to c$ and $d \to a$ are isotherms to prove

$$\frac{V_d}{V_a} = \frac{V_c}{V_b}.$$

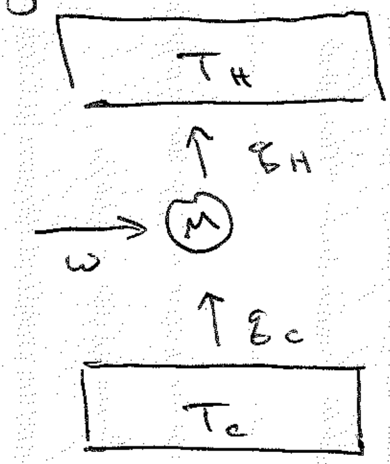
and hence $\boxed{\eta = 1 - \frac{T_C}{T_H}}$

Again from the $2^{nd}$ law a perfect refrigerator, with ~~ΔS=0~~ $q_c = q_H$ and $w=0$ is impossible.

For a real refrigerator we again characterize its performance by

$$\gamma \equiv \frac{\text{what we want}}{\text{what we pay}} = \frac{q_c}{w}$$

**IV** A refrigerator is essentially a heat engine run in reverse




$w + q_c = q_H$

This is

$$\gamma = \frac{q_c}{q_H - q_c}$$

By the $2^{nd}$ law

$$\Delta S = \frac{q_H}{T_H} + \frac{(-q_c)}{T_c} \geq 0$$

$$\Rightarrow \frac{q_c}{q_H} \leq \frac{T_c}{T_H}$$

and so at best

$$\boxed{\gamma = \frac{1}{T_H/T_c - 1} = \frac{T_c}{T_H - T_c}}$$

Carnot for refrigerator