Thermal Physics

Meeting XIII

(There was no Monday meeting due to Fall break)

I Last time

II Overview

III Systems in contact with a heat reservoir

I. • Heat engines

Efficiency

$$\eta \equiv \frac{W}{Q_H} \leq 1 - \frac{T_c}{T_H}$$

All based on cons. of Energy and 2nd law.

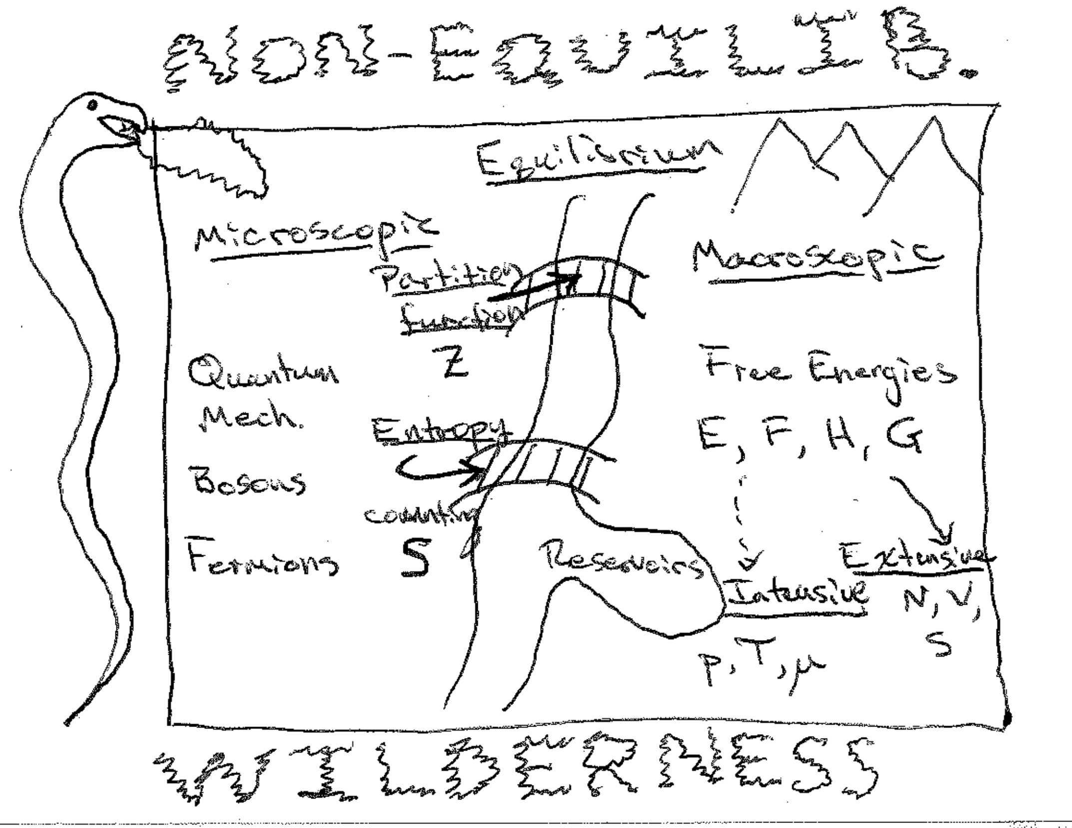
• Refrigerators

Coeff. of performance

$$\gamma \equiv \frac{Q_c}{W} \leq \frac{T_c}{T_H - T_c}$$

Inequalities saturated for Carnot process.

II During the review session I drew the following useful picture, which you should add things to



III Whether we've been very explicit about it or not we've focused on <u>isolated systems</u>. Most often this has occurred through our assumption that we know the energy of the system.
Then our basic postulate ~~was~~ is that all accessible states

How do we picture such systems?
With an ensemble

fix N, V, E

each member is a box of volume V with N particles inside and energy E. Averages are over all the members.
This is called the microcanonical ensemble.

are equally probable, i.e. that

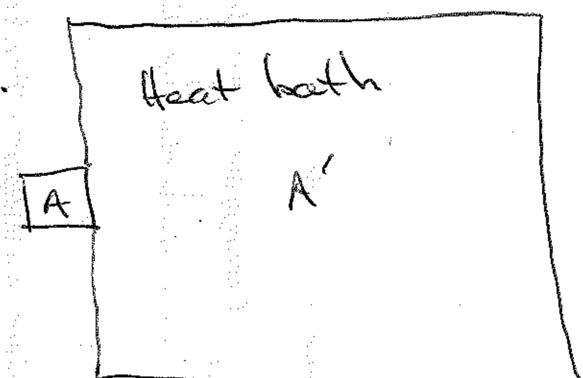
$$P_r = \begin{cases} C & E \le E_r \le E + \delta E \\ 0 & \text{else} \end{cases}$$

where C can be determined through normalization

$$\sum_r P_r = 1$$

with $E \le E_r \le E + \delta E$

Are there alternatives?
Yes!



When these two are in equilibrium what is the probability P_r of finding A in any one particular microstate E_r? Well, again,

$$E^{(0)} = E_r + E'$$

and so $E' = E^{(0)} - E_r$

Since A has definite state r we have

$$P_r = C'\,\Omega'(E^{(0)} - E_r)$$

Then

$$\ln \Omega'(E^{(0)} - E_r) \approx \ln \Omega'(E^{(0)}) - \beta E_r$$

or

$$\Omega'(E^{(0)} - E_r) \approx \Omega'(E^{(0)})\, e^{-\beta E_r}$$

So that

$$\boxed{P_r = C\, e^{-\beta E_r}}$$

We can find C by normalizing

Fine, but if $E_r \ll E^{(0)}$ we can expand

$$\ln \Omega'(E^{(0)} - E_r) = \ln \Omega'(E^{(0)}) - \frac{\partial \ln \Omega'}{\partial E'} E_r + \cdots$$

But A' is a heat bath with definite temperature,

$$\beta = \left.\frac{\partial \ln \Omega'}{\partial E'}\right|_{E_r = 0}$$

$$\sum_r P_r = 1 = C \sum_r e^{-\beta E_r}$$

$$\Rightarrow C = \frac{1}{\sum_r e^{-\beta E_r}}$$

That is,

$$P_r = \frac{e^{-\beta E_r}}{\sum_r e^{-\beta E_r}}$$

The exponential factor

$e^{-\beta E_r}$ is called the "Boltzmann factor" and the distribution is called the Canonical distribution. An ensemble of systems all in contact with a heat reservoir at temp. T is called a canonical ensemble.

$$P(E) = C\,\Omega(E)\,e^{-\beta E}$$

We have

$\Omega(E)$ is rapidly increasing

and

$e^{-\beta E}$ is rapidly decreasing

so

$$P(E) = C\,\Omega(E)\,e^{-\beta E}$$

What about

$P(E)$ = prob of A having energy $\in [E, E+\delta E]$?

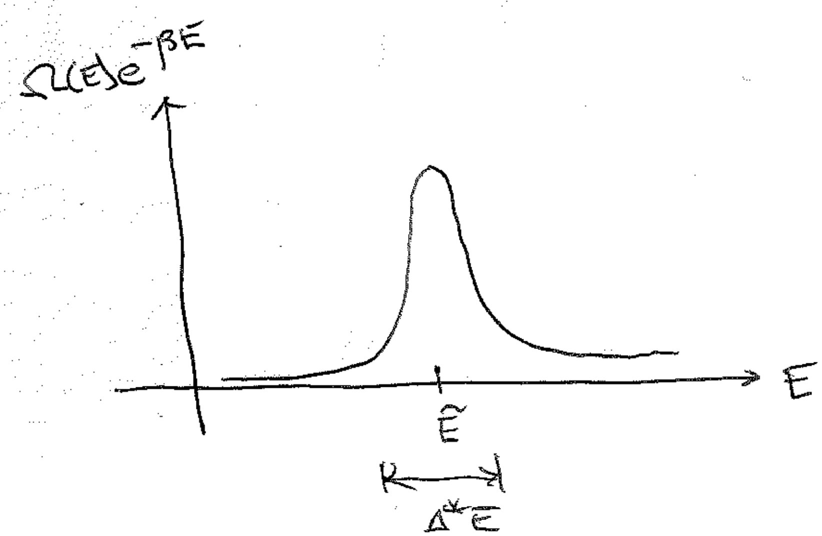
$\cancel{P(E) = C\,\Omega(E)\,e^{-\beta E}}$

First

$$P(E) = \sum_r P_r$$

for states r s.t. $E < E_r < E+\delta E$.

But then



The larger A is the sharper this peak, but again we have the peaked property