Thermal physics

Sep. 2nd, 2014 P 1/5

I Last time

II Prediction logistics & survey results

III Polymers as random walkers

IV Mean & dispersion

~~V Many variables~~

I. 3 most important things we did?

- Simplify counting and build up to hard case
- Reviewed Thermo: heat is energy transfered when ext. params fixed.
- Recalled binomial coeff.

$\binom{52}{5}$ = ways to choose 5 distinct cards out of 52

II Prediction logistics:

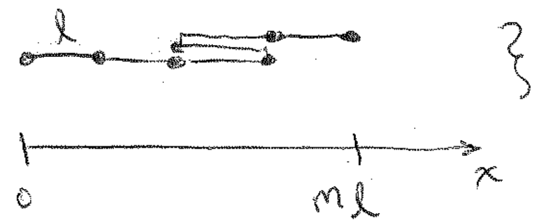
- Accordion folder in corner
- Write xls spreadsheet to do awards calculation. Volunteer?
- Pool bets as we go
- Do awards on Wed.s

Survey results: QM: 3 yes, 5 No

H: 8 yes, Phase Sp.: Heard of yes
a little review yes.

III The most compelling random walker for me is the polymer; physically transparent and enormously applicable.

Chain of monomers, each length l



Assume steps are statistically indep.

p = prob. right step

q = " left "

Then, probability of given sequence is

$$\underbrace{p \cdot p \cdots p}_{n_1} \underbrace{q \cdots q}_{n_2} = p^{n_1} q^{n_2}$$

But there are different ways of doing n_1 right steps, n_2 left. How many?

$$x = ml$$

with $-N \leq m \leq N$

What is $P_N(m)$? Rephrase:

n_1 right steps

n_2 left steps

$$N = n_1 + n_2$$

$$m = n_1 - n_2$$

$$= n_1 - (N - n_1) = 2n_1 - N$$

Ans. $N!/n_1!n_2! = \binom{N}{n_1}$

Thus prob. of n_1 right steps in N steps is,

$$W_N(n_1) = \frac{N!}{n_1!n_2!} p^{n_1} q^{n_2}$$

"Binomial distribution" because of its close relation w/

$$(p+q)^N = \sum_{n=0}^{N} \frac{N!}{n!(N-n)!} p^n q^{N-n}$$

The question we've answered is closely related to the one we want:

$$P_N(m) = W_N(n_1)\,!$$

$$n_1 = \tfrac{1}{2}(N+m), \qquad n_2 = \tfrac{1}{2}(N-m)$$

and hence

$$\boxed{P_N(m) = \frac{N!}{[(N+m)/2]!\,[(N-m)/2]!}\, p^{(N+m)/2}(1-p)^{(N-m)/2}}$$

Do pitching machine practice sheet.

Prediction: Po experimentalists

a) over

IV $u_i \quad i=1,\dots,M;\ P(u_i)$

$$\bar{u} = \sum_i u_i P(u_i)$$

$$\rightsquigarrow \overline{f(u)} = \sum_i f(u_i) P(u_i)$$

Find: $\overline{f(u)+g(u)} = \overline{f(u)} + \overline{g(u)}$

$$\overline{c f(u)} = c\,\overline{f(u)}$$

Pitching suggests we consider precision:

$$\Delta u = u - \bar{u}$$

Average precision?

$$\overline{\Delta u} = \overline{u - \bar{u}} = \bar{u} - \bar{u} = 0$$

Boring. But,

$\overline{\Delta u^2} = $ dispersion of u

is very interesting. Captures how tightly clustered the balls are:

$$\overline{\Delta u^2} = \overline{(u-\bar{u})^2} = \overline{u^2 - 2u\bar{u} + \bar{u}^2}$$

$$= \overline{u^2} - 2\bar{u}^2 + \bar{u}^2 = \overline{u^2} - \bar{u}^2$$

Easy to talk about but how in practice?

Wonderful technique:

$$W(n_1) = \frac{N!}{n_1!(N-n_1)!} p^{n_1} q^{N-n_1}$$

what is $\bar{n}_1$?

$$\bar{n}_1 = \sum_{n_1=0}^{N} n_1 W(n_1)$$

Idea: treat a parameter as a variable

$$\frac{\partial}{\partial p}\left(p^{n_1}\right) = n_1 p^{n_1-1}$$

So,

$$\bar{n}_1 = \sum_{n_1=0}^{N} \frac{N!}{n_1!(N-n_1)!} p \cdot n_1 p^{n_1-1} q^{N-n_1}$$

$$= p \frac{\partial}{\partial p} \sum_{n_1=0}^{N} \frac{N!}{n_1!(N-n_1)!} p^{n_1} q^{N-n_1}$$

$$= p \frac{\partial}{\partial p} (p+q)^N$$

$$= pN(p+q) = \boxed{pN}$$

Use this to your heart's content.

If they are tired/bored have them do dispersion $\overline{(\Delta n_1)^2}$ calculation, if not quote it:

$$\overline{(\Delta n_1)^2} = Npq$$

So,

$$\frac{rms}{\bar{n}_1} = \frac{\sqrt{\overline{(\Delta n_1)^2}}}{\bar{n}_1} = \frac{\sqrt{Npq}}{pN} = \sqrt{\frac{q}{p}} \frac{1}{\sqrt{N}}$$

famous $1/\sqrt{N}$ error fall off.

Look at Fig. 1.4.1 of Reif. What is the binomial distribution? That is, does it look like a function you know?

It's a discrete gaussian! Reif does a careful proof – read it. The parameter technique works here too!

$$\tilde{P}(x) = e^{-ax^2}$$

Normalize: $\int_{-\infty}^{\infty} e^{-ax^2} du = \sqrt{\frac{\pi}{a}}$ ← Memorize!

$$P(x) = \sqrt{\frac{a}{\pi}} e^{-ax^2}$$

$$\bar{x} = \int \underset{\text{even}}{\overset{\text{odd}}{x P(x)}} dx = 0$$ ← Symmetry

For dispersion need,

$$\overline{x^2} = \int x^2\, \wp(x)\, dx$$

$$= \sqrt{\frac{a}{\pi}} \int x^2 e^{-ax^2}$$

$$\frac{d}{da} \int e^{-ax^2} = -\int x^2 e^{-ax^2}$$

$$= \frac{d}{da}\left(\sqrt{\frac{\pi}{a}}\right) = \sqrt{\pi}\left(-\frac{1}{2} a^{-3/2}\right)$$

$\wp(x)\,dx$ is the probability that an event is between x and $x + dx$. For an event btwn x_i and x_f

$$\text{Prob} = \int_{x_i}^{x_f} \wp(x)\, dx.$$

Intuition: Needs a different symbol because if x carries units $\wp(x)$ has to cancel them out in

so,

$$\overline{x^2} = \sqrt{\frac{a}{\pi}} \cdot \sqrt{\pi}\, \frac{1}{2}\, a^{-3/2}$$

$$= \boxed{\frac{1}{2a}}.$$

Why $\wp$? "Probability density"

~~An intuitive perspective: What if x carries units?~~ Integrate to find probability or

the integral. E.g. x = position then $[\wp(x)] = \frac{1}{\text{position}}$.

This is the sense in which it is a density.

~~V Many variables and functions~~

~~u_i $i = 1, \ldots, M$~~

~~v_j $j = 1, \ldots, N$~~

~~$P(u_i, v_j)$~~