# Thermal Physics



I Last time

II Calculating the Fermi energy $\mu$

III Heat capacity of Metals

IV Bose - Einstein Statistics

I. • Showed

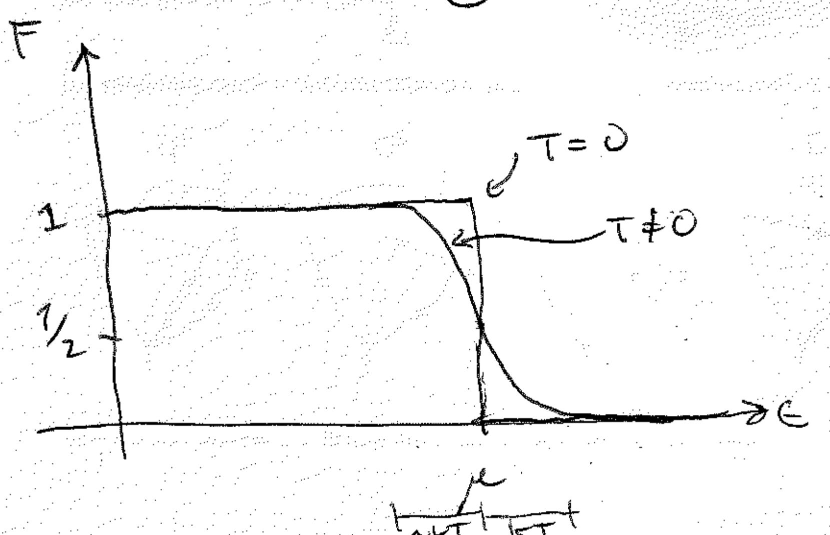
$$\bar{n}_s = \frac{\sum_{n_s} n_s e^{-\beta \epsilon_s n_s} \sum^{(s)}_{n_1, n_2, \ldots} e^{-\beta(n_1\epsilon_1 + n_2\epsilon_2 + \cdots)}}{\sum_{n_s} e^{-\beta n_s \epsilon_s} \sum^{(s)}_{n_1, n_2, \ldots} e^{-\beta(n_1\epsilon_1 + n_2\epsilon_2 + \cdots)}}$$

and derived the Fermi-Dirac distribution

$$\bar{n}_s = \frac{1}{e^{\beta(\epsilon_s - \mu)} + 1}$$

• Studied the Fermi function

$$F(\epsilon) = \frac{1}{e^{\beta(\epsilon - \mu)} + 1}$$




II At $T=0$, what is $\mu_0$? ← $T=0$ Fermi energy

Each particle has

$$\epsilon = \frac{p^2}{2m} = \frac{\hbar^2 k^2}{2m}$$

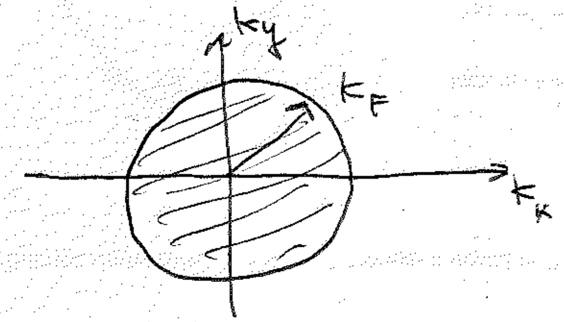
The Fermi energy corresponds to a Fermi momentum

$$\mu_0 = \frac{p_F^2}{2m} = \frac{\hbar^2 k_F^2}{2m}.$$

We know already that it is easiest to count states in phase space — when the position dependence is trivial, this means in momentum Space. ~~So~~ To relate $\mu_0$ to the # of particles N, we need to count states.

the density of states in wave-vector Space.

Ok, from above, all the states up to $k_F$ are filled




For a spatial Volume V we have

$$\# \text{ of States} = \frac{V \cdot d^3\vec{p}}{h^3}$$

$$= \frac{V \cdot d^3\vec{p}}{(2\pi)^3 \hbar^3} = \frac{V}{(2\pi)^3} d^3\vec{k}$$

$$= \rho \, d^3\vec{k}$$

where we have introduced

$$\rho = \frac{V}{(2\pi)^3}$$

$$\underset{\#\text{ particles}}{N} = \underset{\text{two spins}}{2} \cdot \iiint^{k_F} \frac{V}{(2\pi)^3} d^3\vec{k}$$

$$= 2 \cdot \frac{V}{(2\pi)^3} \frac{4}{3}\pi k_F^3$$

or

$$N = \frac{V}{3\pi^2} k_F^3$$

Then

$$\boxed{k_F = \left(3\pi^2 \frac{N}{V}\right)^{1/3}}$$

Then

$$\mu_0 = \frac{\hbar^2}{2m} k_F^2 = \frac{\hbar^2}{2M}\left(3\pi^2 \frac{N}{V}\right)^{2/3}$$

III Many electrons have $\epsilon \ll \mu$ and have little effect on $C_V$.

For a classical gas (MB statistics)

$$C_V = \left(\frac{\partial \bar{E}}{\partial T}\right)_V$$

gives

$$\bar{E} = \frac{3}{2} NkT$$

and

$$C_V = \frac{3}{2} Nk.$$

But electrons with $\epsilon \ll \mu$ don't respond much to a change in $T$. There is a small # of electrons,

$N_{eff}$, that are ~~affected~~ in band $\sim kT$ around $\mu$. These give roughly $\frac{3}{2}k$ each because for $\epsilon > \mu$ $F \propto e^{-\beta\epsilon}$ like MB.

How many? Let $\underbrace{\rho(\epsilon)d\epsilon}_{\text{\# of states}}$ (density of states) in $\epsilon$ to $\epsilon + d\epsilon$ and get

$$N_{eff} \approx \rho(\mu) kT$$

So that,

$$C_V \approx N_{eff}\left(\frac{3}{2}k\right)$$

$$= \frac{3}{2} k^2 \rho(\mu) T$$

or even more roughly

$$N_{eff} \approx \left(\frac{kT}{\mu}\right) N$$

and

$$C_V \approx \frac{3}{2} Nk \frac{kT}{\mu}$$

$$= \frac{3}{2} Nk \frac{T}{T_f}, \quad T_f \equiv \frac{\mu}{k}$$

For copper

$$T_F \equiv \frac{\mu_0}{k} \approx 80{,}000\,^\circ K$$

and this contribution is negligible compared to that of the phonons (lattice vibrations). Before quantum it was hard to explain why the electrons didn't contribute

so,

$$\bar{n}_s = \frac{0 + e^{-\beta\epsilon_s} Z_s(N-1) + 2e^{-2\beta\epsilon_s} Z_s(N-2) + \cdots}{Z_s(N) + e^{-\beta\epsilon_s} Z_s(N-1) + e^{-2\beta\epsilon_s} Z_s(N-2) + \cdots}$$

Recall,

$$Z_s(N-\Delta N) \approx e^{-\alpha \Delta N} Z_s(N)$$

so

$$\bar{n}_s = \frac{Z_s(N)\left[0 + e^{-\beta\epsilon_s} e^{-\alpha} + 2e^{-2\beta\epsilon_s} e^{-2\alpha} + \cdots\right]}{Z_s(N)\left[1 + e^{-\beta\epsilon_s} e^{-\alpha} + e^{-2\beta\epsilon_s} e^{-2\alpha} + \cdots\right]}$$

an extra $\frac{3}{2}R$ over the phonons.

IV Similar to our FD derivation.

$$\bar{n}_s = \frac{\sum_{n_s} n_s e^{-\beta\epsilon_s n_s} \sum^{(s)} e^{-\beta(\epsilon_1 n_1 + \cdots)}}{\sum_{n_s} e^{-\beta\epsilon_s n_s} \sum^{(s)} e^{-\beta(\epsilon_1 n_1 + \cdots)}}$$

or

$$\bar{n}_s = \frac{\sum_{n_s} n_s e^{-n_s(\alpha+\beta\epsilon_s)}}{\sum_{n_s} e^{-n_s(\alpha+\beta\epsilon_s)}}$$

$$= -\frac{1}{\beta}\frac{\partial}{\partial\epsilon_s} \ln \sum_{n_s} e^{-n_s(\alpha+\beta\epsilon_s)}$$

$$= -\frac{1}{\beta}\frac{\partial}{\partial\epsilon_s} \ln \frac{1}{1 - e^{-(\alpha+\beta\epsilon_s)}}$$

$$= \frac{1}{\beta}\frac{1}{1-e^{-(\alpha+\beta\epsilon_s)}} \cdot \beta e^{-(\alpha+\beta\epsilon_s)}$$

That is

$$\bar{n}_s = \frac{1}{e^{\alpha+\beta\epsilon_s} - 1}$$

$$= \boxed{\frac{1}{e^{\beta(\epsilon_s-\mu)} - 1}} \text{ BE stats.}$$

We will start studying Bose-Einstein condensation next time.