Thermal Physics

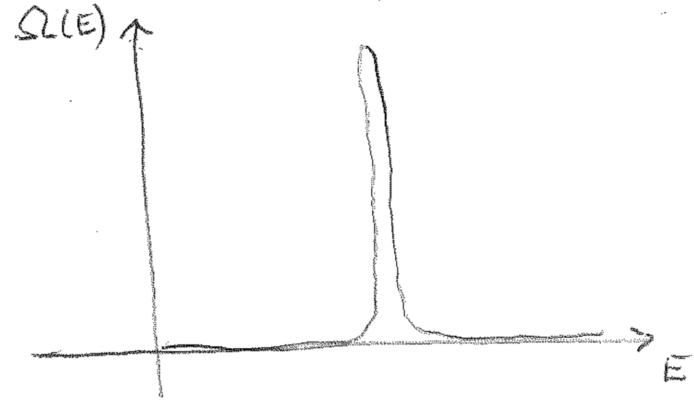
Meeting VI

I Last time

II A little on Temperature

III Heat reservoirs

I • Sharpness of the # of states and prob.



• Density of states

$$\Omega(E) = \omega(E)\,\delta E$$

• Entropy and Temperature

$$S = k \ln \Omega$$

$$\beta = kT$$

at Equilibrium

$$T = T'$$

and

$$S + S' = \text{maximum.}$$

II Recall that we defined

$$\frac{1}{kT} \equiv \beta \equiv \frac{\partial \ln \Omega}{\partial E}$$

We have also argued that generically Ω is a very rapidly increasing function of energy. So then, generally,

$\beta > 0$ and $T > 0$

In particular

$$\Omega(E) \propto E^f$$

So,

$$\ln\Omega \approx f \ln E + \text{const.}$$

and at Equilib.

$$\beta = \frac{\partial \ln\Omega}{\partial E} = \frac{f}{\tilde{E}} = \frac{f}{E}$$

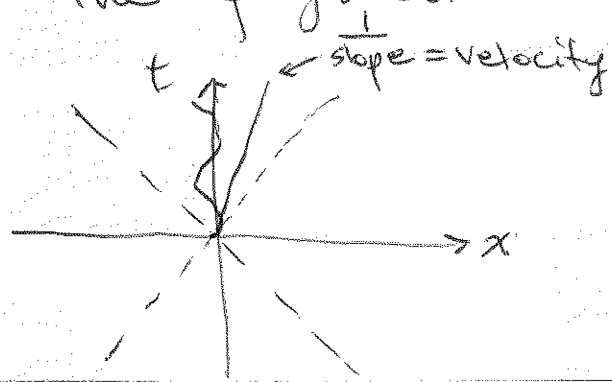
or

$$\boxed{kT \approx \frac{E}{f}}$$

Temperature is a measure of roughly how much energy each degree of freedom is carrying.

What is the physical meaning of k? Every physicist gives a different answer — many say that it has none and is an unfortunate historical accident. I disagree.

What is the physical meaning of c?



It provides us the limiting speed for information transfer.

$\hbar$ provides us the smallest area in phase space.

What happens to T when we double the amount of stuff in a sample?

$$\square + \square = \boxed{}$$

Absolutely nothing! (Statement of equilibrium.) T is what we call an intensive variable. But the energy certainly doubles! If T measures some feature of the energy, how are these two statements consistent?

It gives the dimensionless f physical like, i.e. life in terms of units. Write it

$$T = \frac{\bar{E}}{(fK)}$$

to make this explicit.

III Heat reservoir or heat bath:

A' is a heat reservoir (or bath)

The resolution is, of course, that the # of d.o.f also doubles when you double the system. The constant K reminds us that you need discrete constituents for temp. to be meaningful!

If its temp. doesn't change much in interacting with system A

$$\left| \frac{\partial \beta'}{\partial E'} Q' \right| \ll \beta'.$$

Now, suppose A' absorbs Q' then

$$\ln \Omega'(E'+Q') - \ln \Omega'(E')$$
$$= \frac{\partial \ln \Omega'}{\partial E'} Q' + \frac{1}{2} \frac{\partial^2 \ln \Omega'}{\partial E'^2} Q'^2 + \dots$$

$$= \beta' Q' + \frac{1}{2} \frac{\partial \beta'}{\partial E'} Q'^2 + \cdots$$

by assumption the 2nd and higher terms are small (heat bath), so that

$$\ln \Omega'(E'+Q') - \ln \Omega'(E') = \beta' Q'$$

or

$$\boxed{\Delta S' = \frac{Q'}{T'}}$$ for heat reservoir

More generally, it is always true that we can neglect the higher order terms when $đQ$ is infinitessimal, then

$$\boxed{dS = \frac{đQ}{T}}.$$

IV Solved problems 3.1 and 3.2 in Reif.