Thermal Physics

Meeting VIII

I Last time

II Macroscopic determination of ~~quantities~~ $W, \bar{E}, Q, T$.

III Heat capacity and specific heat

IV Macro determination of entropy

I

$$\boxed{kT \approx \frac{\bar{E}}{f}}$$

$$\boxed{\frac{\Delta^* E}{\bar{E}} = \frac{1}{\sqrt{f}}}$$

$$\boxed{\bar{X} = \frac{1}{\beta}\frac{\partial \ln \Omega}{\partial x}} \qquad \boxed{\beta = \frac{\partial \ln \Omega}{\partial E}}$$

Equations of state.

II If we only have access to macroscopic parameters how do we measure them? And how do we characterize a system with them?

Work: If a quasi-static process takes V_i to V_f then

$$W = \int_{V_i}^{V_f} \bar{p}(V)\, dV$$

Work done by system

Internal Energy Thermally insulate. Then $Q = 0$ and

$$\Delta \bar{E} = -W$$

or

$$\bar{E}_b - \bar{E}_a = -W_{ab} = -\int_a^b đW$$

As always we are free to choose our zero of energy. For convenience choose experimentally easy state $\bar{E}_a = 0$. Then

$$\bar{E}_b = -W_{ab}$$

Heat: We have also shown

$$Q_{ab} = \Delta\bar{E} + W_{ab}$$
$$= \underset{\text{measure}}{\bar{E}_b} - \bar{E}_a + \underset{\text{measured}}{W_{ab}}$$

$\Rightarrow$ gives Q_{ab}.

Various ways of achieving this in practice — important but more practical to study in context.

Keeping V fixed and measuring $\bar{p}$ gives you access to T.

III Not all systems respond to having heat added in the same way — feet on cold tile vs. feet on carpet.

The heat capacity is a way to capture these differences

$$(đQ)_y = (C_y\, dT)_y$$

(Absolute) Temperature:

Using an equation of state

[Recall: Equations of state relate generalized forces, external params, and temperature. Get 'em from Ω,

$$\beta = \frac{\partial \ln\Omega}{\partial E} \qquad \bar{X}_\alpha = \frac{1}{\beta}\frac{\partial \ln\Omega}{\partial x_\alpha}$$]

$$\bar{p} = \frac{1}{\beta}\frac{\partial \ln\Omega}{\partial V} \Rightarrow \bar{p}V = NkT$$

or

$$C_y \equiv \left(\frac{đQ}{dT}\right)_y \leftarrow \text{at constant } y$$

as such

$$C_y = C_y(T, y).$$

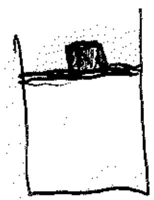
Of course, if you have more stuff it can absorb more heat. This leads to

$$c_y \equiv \frac{1}{\nu} C_y = \frac{1}{\nu}\left(\frac{đQ}{dT}\right)_y$$

and

$$c_y' \equiv \frac{1}{m} C_y = \frac{1}{m}\left(\frac{đQ}{dT}\right)_y$$

The dependence on y is significant, Common case $y = V$ or $y = p$



For c_V we fix the piston, then

$$đQ = d\bar{E} \quad \text{and} \quad c_V = \frac{1}{\nu}\left(\frac{\partial \bar{E}}{\partial T}\right)_V$$

Consider a copper block of mass m_A immersed in a tank of water of mass M_B at 1 atm of pressure. Suppose that c_A' and c_B' are given and for the temperatures of this setup $c_A'(T) = \text{const}$ (small c), $c_B' = \text{const}$; these temps are T_A and T_B. What is T_f?

For c_p in general

$$đQ = d\bar{E} + \bar{p}\, dV$$

both internal energy increases and work is performed. This means that for a given $đQ$, $d\bar{E}$ changes less and hence dT is smaller. This means

$$c_p > c_V$$

Well,

$$Q_A + Q_B = 0.$$

Now

$$đQ = m c' dT$$

So

$$Q_A = m_A \int_{T_A}^{T_f} c_A'\, dT = m_A c_A' (T_f - T_A)$$

and similarly

$$Q_B = m_B c_B' (T_f - T_B)$$

So

$$(m_B c_B' + m_A c_A') T_f = m_A c_A' T_A + m_B c_B' T_B$$

or

$$\boxed{T_f = \frac{m_A c_A' T_A + m_B c_B' T_B}{(m_B c_B' + m_A c_A')}}$$

Finally, we can express heat capacities in terms of entropy

$$đQ = T\,dS$$

so

$$C_y = T\left(\frac{\partial S}{\partial T}\right)_y$$

In our previous example, what is the total entropy change?

$$\Delta S_A = S_A(T_f) - S_A(T_A) = \int_{T_A}^{T_f} \frac{m_A c_A'\,dT}{T}$$

$$= m_A c_A' \ln\left(\frac{T_f}{T_A}\right)$$

Similarly for ΔS_B and so

$$\Delta S_A + \Delta S_B = m_A c_A' \ln\frac{T_f}{T_A} + m_B c_B' \ln\frac{T_f}{T_B}$$

IV Entropy

$$dS = \frac{đQ}{T}$$

so

$$S_b - S_a = \int_a^b \frac{đQ}{T} \quad \left(\begin{array}{c}\text{quasi-}\\ \text{static}\\ \text{process}\end{array}\right)$$

If we've already measured the heat capacity then

$$S(T_b) - S(T_a) = \int_a^b \frac{đQ}{T} = \int_{T_a}^{T_b} \frac{C_y(T')\,dT'}{T'}$$

A nifty argument shows that

$$\Delta S_A + \Delta S_B \geq 0$$

First $\ln x \leq x - 1$

$$\Rightarrow -\ln x \geq -x + 1$$

$$\Rightarrow \ln\frac{1}{x} \geq -x + 1$$

Let $y = \frac{1}{x}$ $\quad \ln y \geq -\frac{1}{y} + 1$

$$\Delta S_A + \Delta S_B \geq m_A c_A'\left(1 - \frac{T_A}{T_f}\right) + m_B c_B'\left(1 - \frac{T_B}{T_f}\right)$$

$$= \frac{1}{T_f}\left[m_A c_A'(T_f - T_A) + m_B c_B'(T_f - T_B)\right] = 0.$$