Homework 1

Due by 10pm on Wednesday, September 9th, 2020

Reading: Schroeder Chap. 1. Class notes.

1. The Fahrenheit temperature scale is defined so that ice melts at 32°F and water boils at 212°F.
(a) Derive the formulas for converting from Fahrenheit to Celsius and back.
(b) What is absolute zero on the Fahrenheit scale?
(c) How do you convert between the Celsius and kelvin scales? The Fahrenheit and kelvin scales?
(d) Why are many of the formulas that we will use in this course only correct when using the kelvin scale?

2. Determine the kelvin temperature for each of the following:
(a) human body temperature;
(b) the boiling point of water (at the standard pressure of 1 atm);
(c) the coldest day you can remember;
(d) the boiling point of liquid nitrogen (196°C);
(e) the melting point of lead (327°C).

3. For a solid, we define the linear thermal expansion coefficient, α, as the fractional increase in length per degree:

$$\alpha \equiv \frac{\Delta L/L}{\Delta T}.$$

(a) For steel, α is 1.1×10^{-5} K^{-1}. Estimate the total variation in length of a 1-km steel bridge between a cold winter night and a hot summer day.
(b) The dial thermometer in Schroeder's Figure 1.2 uses a coiled metal strip made of two different metals laminated together. Explain how this works.
(c) Prove that the volume thermal expansion coefficient of a solid (see Schroeder Problem 1.7) is equal to the sum of its linear expansion coefficients in the three directions: ($\beta = \alpha_x + \alpha_y + \alpha_z$. (So for an isotropic solid, which expands the same in all directions, $\beta = 3\alpha$.)

4. A mole is approximately the number of protons in a gram of protons. The mass of a neutron is about the same as the mass of a proton, while the mass of an electron is usually negligible in comparison, so if you know the total number of protons and neutrons in a molecule (i.e., its "atomic mass"), you know the approximate mass (in grams) of a mole of these molecules.[1] Referring to the periodic table (e.g. at the back of the class text), find the mass of a mole of each of the following: water, nitrogen (N_2), lead, quartz (SiO_2).

5. (a) What is the volume of one mole of air, at room temperature and 1 atm pressure?
(b) Estimate the number of air molecules in an average-sized room.
(c) Calculate the mass of a mole of dry air, which is a mixture of N_2 (78% by volume), O_2 (21%), and argon (1%).

[1] The precise definition of a mole is the number of carbon-12 atoms in 12 grams of carbon-12. The **atomic mass** of a substance is then the mass, in grams, of exactly one mole of that substance. Masses of individual atoms and molecules are often given in **atomic mass units**, abbreviated "u", where 1 u is defined as exactly 1/12 the mass of a carbon-12 atom. The mass of an isolated proton is actually slightly greater than 1 u, while the mass of an isolated neutron is slightly greater still. But in this problem, as in most thermal physics calculations, it's fine to round atomic masses to the nearest integer, which amounts to counting the total number of protons and neutrons.

6. The exponential atmosphere.
(a) Consider a horizontal slab of air whose thickness (height) is dz. If this slab is at rest, the pressure holding it up from below must balance both the pressure from above and the weight of the slab. Use this fact to find an expression for dP/dz, the variation of pressure with altitude, in terms of the density of air.
(b) Use the ideal gas law to write the density of air in terms of pressure, temperature, and the average mass m of the air molecules. (The information needed to calculate m is given in 4.(c).) Show, then, that the pressure obeys the differential equation

$$\frac{dP}{dz} = -\frac{mg}{kT}P,$$

called the **barometric equation**. (c) Assuming that the temperature of the atmosphere is independent of height (not a great assumption but not terrible either), solve the barometric equation to obtain the pressure as a function of height: $P(z) = P(0)e^{-mgz/kT}$. Show also that the density obeys a similar equation.

7. Go to the Computing tab of our course website and download the "Instructions for Python and Jupyter 1: Lists, Plotting, and Fitting" and "The Lists, Plotting, and Fitting notebook." For many of you this notebook will be purely review, but that's fine. For this problem I want you to apply these tools. Referencing Figure 1.3 of Schroeder, do your best to accurately estimate all of the data points on this graph. Enter these data in your Jupyter notebook and use linear fits to find the slopes of the three lines. In your solutions for this problem include: a screen shot of a plot containing your best fit lines and the slopes of each line. Using these slopes, estimate by what amount the gas in the fixed volume was changed for each of the three experiments. Also, include a brief explanation of how you've obtained these estimates and their values.