Homework 2

Due by 10pm on Wednesday, September 16th, 2020

Reading: Schroeder Chap. 2, sections 2.1-2. Class notes.

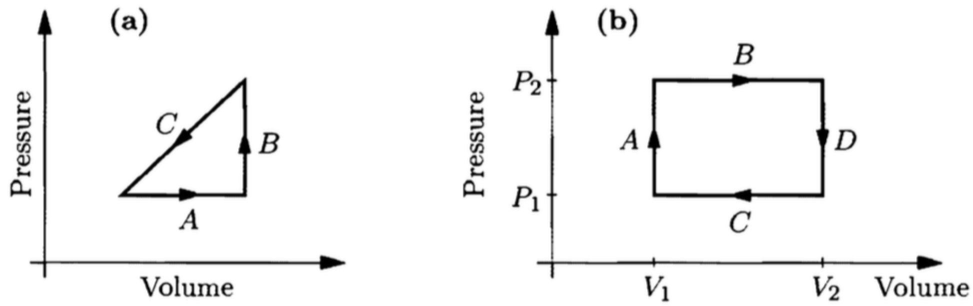

Figure 1: PV diagrams for the processes described in problem 1.

1. An ideal gas is made to undergo the cyclic process shown in panel (a) of the Figure. For each of the steps A, B, and C, determine whether each of the following is positive, negative, or zero:
(a) the work done on the gas;
(b) the change in the energy content of the gas;
(c) the heat added to the gas. Then determine the sign of each of these three quantities for the whole cycle. What does this process accomplish?
An ideal diatomic gas, in a cylinder with a movable piston, undergoes the rectangular cyclic process shown in panel (b) of the Figure. Assume that the temperature is always such that rotational degrees of freedom are active, but vibrational modes are "frozen out." Also assume that the only type of work done on the gas is quasistatic compression-expansion work.
(d) For each of the four steps A through D, compute the work done on the gas, the heat added to the gas, and the change in the energy content of the gas. Express all answers in terms of P_1, P_2, V_1, and V_2. (Hint: Compute ΔU before Q, using the ideal gas law and the equipartition theorem.)
(e) Describe in words what is physically being done during each of the four steps; for example, during step A, heat is added to the gas (from an external flame or something) while the piston is held fixed.
(f) Compute the net work done on the gas, the net heat added to the gas, and the net change in the energy of the gas during the entire cycle. Are the results as you expected? Explain briefly.

2. (a) Derive Schroeder's Eq. (1.40) from his Eq. (1.39).
In the course of pumping up a bicycle tire, a liter of air at atmospheric pressure is compressed adiabatically to a pressure of 7 atm. (Air is mostly diatomic nitrogen and oxygen.)
(b) What is the final volume of this air after compression?
(c) How much work is done in compressing the air?
(d) If the temperature of the air is initially 300 K, what is the temperature after compression?
(e) In a Diesel engine, atmospheric air is quickly compressed to about 1/20 of its original volume. Estimate the temperature of the air after compression, and explain why a Diesel engine does not require spark plugs.

3. Two identical bubbles of gas form at the bottom of a lake, then rise to the surface. Because the pressure is much lower at the surface than at the bottom, both bubbles expand as they rise. However, bubble A rises very quickly, so that no heat is exchanged between it and the water. Meanwhile, bubble B rises slowly (impeded by a tangle of seaweed), so that it always remains in thermal equilibrium with the water (which has the same temperature everywhere). Which of the two bubbles is larger by the time they reach the surface? Explain your reasoning fully.

4. **Speed of Sound**—By applying Newton's laws to the oscillations of a continuous medium, one can show that the speed of a sound wave is given by

$$c_s = \sqrt{\frac{B}{\rho}},$$

where ρ is the density of the medium (mass per unit volume) and B is the **bulk modulus**, a measure of the medium's stiffness. More precisely, if we imagine applying an increase in pressure ΔP to a chunk of the material, and this increase results in a (negative) change in volume ΔV, then B is defined as the change in pressure divided by the magnitude of the fractional change in volume:

$$B \equiv \frac{\Delta P}{-\Delta V/V}.$$

This definition is still ambiguous, however, because I haven't said whether the compression is to take place isothermally or adiabatically (or in some other way).
(a) Compute the bulk modulus of an ideal gas, in terms of its pressure P, for both isothermal and adiabatic compressions.
(b) Argue that for purposes of computing the speed of a sound wave, the adiabatic B is the one we should use.
(c) Derive an expression for the speed of sound in an ideal gas, in terms of its temperature and average molecular mass. Compare your result to the formula for the rms speed of the molecules in the gas. Evaluate the speed of sound numerically for air at room temperature.
(d) When Scotland's Battlefield Band played in Utah, one musician remarked that the high altitude threw their bagpipes out of tune. Would you expect altitude to affect the speed of sound (and hence the frequencies of the standing waves in the pipes)? If so, in which direction? If not, why not?

5. To measure the heat capacity of an object, all you usually have to do is put it in thermal contact with another object whose heat capacity you know. As an example, suppose that a chunk of metal is immersed in boiling water (100° C), then is quickly transferred into a Styrofoam cup containing 250 g of water at 20° C. After a minute or so, the temperature of the contents of the cup is 24°C. Assume that during this time no significant energy is transferred between the contents of the cup and the surroundings. The heat capacity of the cup itself is negligible.
(a) How much heat is gained by the water?
(b) How much heat is lost by the metal?
(c) What is the heat capacity of this chunk of metal?
(d) If the mass of the chunk of metal is 100 g, what is its specific heat capacity?

6. **Dry Adiabatic Lapse Rate**—On the last homework you calculated the pressure of earth's atmosphere as a function of altitude, assuming constant temperature. Ordinarily, however, the

temperature of the bottommost 10-15 km of the atmosphere (called the troposphere) decreases with increasing altitude, due to heating from the ground (which is warmed by sunlight). If the temperature gradient $|dT/dz|$ exceeds a certain critical value, convection will occur: Warm, low-density air will rise, while cool, high-density air sinks. The decrease of pressure with altitude causes a rising air mass to expand adiabatically and thus to cool. The condition for convection to occur is that the rising air mass must remain warmer than the surrounding air despite this adiabatic cooling.
(a) Show that when an ideal gas expands adiabatically, the temperature and pressure are related by the differential equation

$$\frac{dT}{dP} = \frac{2}{f+2}\frac{T}{P}.$$

(b) Assume that dT/dz is just at the critical value for convection to begin, so that the vertical forces on a convecting air mass are always approximately in balance. Use the result from Hw1 6.(b) to find a formula for dT/dz in this case. The result should be a constant, independent of temperature and pressure, which evaluates to approximately $-10°$ C/km. This fundamental meteorological quantity is known as the **dry adiabatic lapse rate**.

7. **Real (Non-Ideal) Gases**—Even at low density, real gases don't quite obey the ideal gas law. A systematic way to account for deviations from ideal behavior is the virial expansion,

$$PV = nRT\left(1 + \frac{B(T)}{(V/n)} + \frac{C(T)}{(V/n)^2} + \cdots\right),$$

where the functions $B(T), C(T)$, and so on are called the **virial coefficients**. When the density of the gas is fairly low, so that the volume per mole is large, each term in the series is much smaller than the one before. In many situations it's sufficient to omit the third term and concentrate on the second, whose coefficient $B(T)$ is called the second virial coefficient (the first coefficient being 1). Here are some measured values of the second virial coefficient for nitrogen (N_2):

$T(K)$	B (cm^3/mol) of N_2
100	-160
200	-35
300	-4.2
400	9.0
500	16.9
600	21.3

Download and run through the very short "Instructions for Python and Jupyter 2: Defining functions and interacting with graphics notebook" from the computing tab of our website.

(a) Write a python code that, given a temperature and virial coefficient B, computes the second term in the virial equation, $B(T)/(V/n)$. Use this code and the table to compute the second term for nitrogen at atmospheric pressure. Discuss the validity of the ideal gas law under these conditions.
(b) Think about the forces between molecules, and explain why we might expect $B(T)$ to be negative at low temperatures but positive at high temperatures.
(c) Any proposed relation between P, V, and T, like the ideal gas law or the virial equation, is

called an equation of state. Another famous equation of state, which is qualitatively accurate even for dense fluids, is the van der Waals equation,

$$\left(P + \frac{an^2}{V^2}\right)(V - nb) = nRT,$$

where a and b are constants that depend on the type of gas. Calculate the second and third virial coefficients (B and C) for a gas obeying the van der Waals equation, in terms of a and b. (Hint: The binomial expansion says that $(1+x)^p \approx 1 + px + \frac{1}{2}p(p-1)x^2$, provided that $|px| \ll 1$. Apply this approximation to the quantity $[1 - (nb/V)]^{-1}$.)
(d) Use python to plot a graph of the van der Waals prediction for $B(T)$, choosing a and b so as to approximately match the data given above for nitrogen. Discuss the accuracy of the van der Waals equation over this range of conditions. (The van der Waals equation is discussed much further in Section 5.3 of our Schroeder text and will be very useful when you get into Molecular Dynamics simulations.)