Today

I. Last Time
II. Remaining logistics
III. Equations of State
IV. Kinetics, Equipartition, Temperature

The main theme of our course is going to be connecting the microscopic world to the macroscopic world we see around us.

I. Defs: Temperature is the thing a thermometer measures. An extensive variable scales linearly with the size of the system. An intensive variable is invariant when you scale the size of the system.

Equilibrium is what happens after two systems have been in contact for some time and no longer spontaneously changing.
We discussed how macroscopic observables emerge from spatial and time averages.

III. Equations of State

At the end of class we discussed an idea introduced by Gibbs, the Fundamental Equation of thermodynamics:

$$f(S, U, V, N) = 0.$$

An example of the fundamental equation for a gas is the Sackur-Tetrode equation:

$$S = Nk\left[\ln\left(\frac{V}{N}\left(\frac{4\pi mU}{3Nh^2}\right)^{3/2}\right) + \frac{5}{2}\right].$$

The time scale that it takes a system to reach equilibrium after being disturbed is called the "relaxation time".

As we've said a couple of times it can be difficult to find the fundamental equation, both theoretically and empirically. So we need alternatives.

III. Equations of State

The most common kind of alternative is an equation of state: an example is the ideal gas law

$$PV = NkT = nRT.$$

Here P is the pressure, V is the volume, N is the number of particles in the gas, T is its temperature, and k is Boltzmann's constant

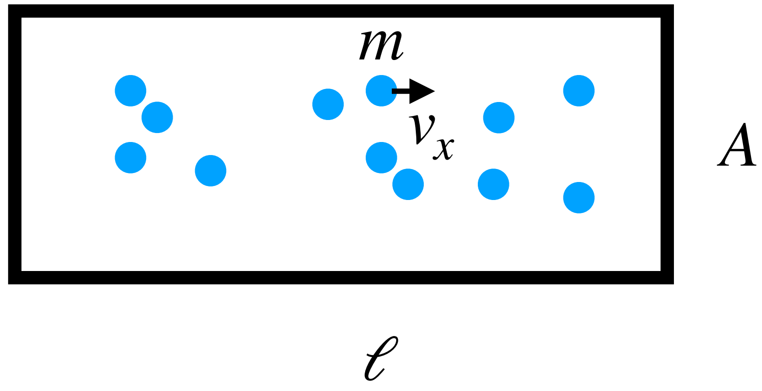
$$k = 1.38 \times 10^{-23}\,\mathrm{J/K}.$$

In the second form n is the number of "moles", and R is called the ideal gas constant. The ideal gas constant is

$$R = 8.31\,\mathrm{J/(mol{*}K)}.$$

We like equations of state because they are more easily measured! A wonderful example of this is the recent collisions between neutron stars that LIGO-Virgo have measured.

IV. Kinetics, Equipartition, and Temperature



What is pressure? In a collision with the right wall, what is momentum delivered to the wall?

$$\Delta p = 2mv_x.$$

The time between subsequent collisions is

$$\Delta t = \frac{2\ell}{v_x}.$$

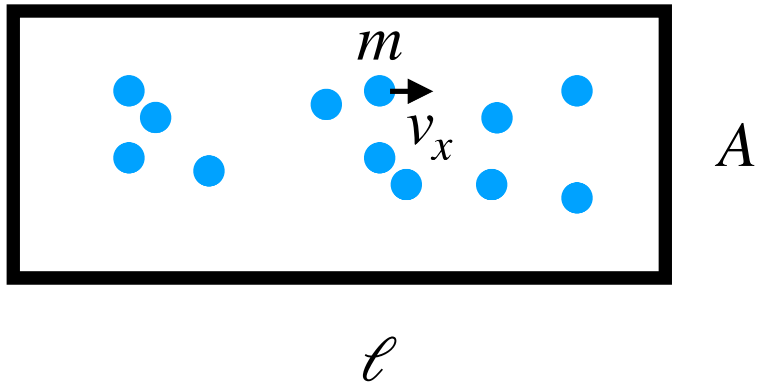
This gives me the average force:

$$F = \frac{\Delta p}{\Delta t} = \frac{2mv_x}{2\ell}v_x = \frac{1}{\ell}mv_x^2.$$

Then the pressure is

$$P = \frac{F}{A} = \frac{1}{A \cdot \ell}mv_x^2 = \frac{1}{V}mv_x^2.$$

IV. Kinetics, Equipartition, and Temperature



Then the pressure is

$$P = \frac{F}{A} = \frac{1}{A \cdot \ell} m v_x^2 = \frac{1}{V} m v_x^2.$$

This was for one particle. For N molecules,

$$PV = Nm\overline{v_x^2}.$$

On average all of

$$\overline{v_x^2}, \overline{v_y^2}, \text{ and } \overline{v_z^2},$$

should be the same. Then

$$PV = \frac{1}{3} Nm\overline{v^2}$$

because $v^2 = v_x^2 + v_y^2 + v_z^2$. Now we use the empirical ideal gas law

$$NkT = \frac{1}{3} Nm\overline{v^2} \implies T = \frac{2}{3}\frac{\overline{E}}{k}.$$