Today

I. Questions from the exam?
II. Last Time
III. Multiplicities and Entropy
IV. Classical Mechanics, Quantum Mechanics, and Entropy

I. More discussion of the exam in our homework sessions.
II. Julia was teaching us how to count. She introduced us to:

The choose function: $\binom{p}{q} = \frac{p!}{(p-q)!q!}$,

which counts the number of ways of drawing q things from p total options, where we only care about the final combination.

Introduced the Einstein solid: a collection of N harmonic oscillators containing q units of energy.

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which counts the number of ways of drawing q things from p total options, where we only care about the final combination.

Introduced the Einstein solid: a collection of N harmonic oscillators containing q units of energy. Showed us how to compute the multiplicity:

$$\Omega(N, q) = \binom{N+q-1}{q}.$$

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A macrostate is the specification of the macroscopic properties of the system. In the Einstein solid this is just N and q.

A microstate is the complete of all the microscopic degrees of freedom of the system.

Then, the multiplicity of system is the number of microstates associated to a particular macrostate: $\Omega = \Omega(N, q)$ is the number of microstates associated to the macrostate (N, q).

Boltzmann defined the <u>entropy of a macrostate</u> as the logarithm of its multiplicity:

$$S = k \ln \Omega.$$

III. Classical Mechanics

Newton: $a = \frac{d^2x}{dt^2} = \frac{F(x)}{m}$ 2nd order ODE, Cartesian coord.s, initial data $(x(0), \dot{x}(0))$.

Lagrange: $\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}\right) = \frac{\partial L}{\partial q}$ 2nd order ODE, general coord.s, here $L \equiv T - U$, initial data $(q(0), \dot{q}(0))$

Huge advantages! General coords are very adaptable, a single scalar determines all the equations of motion.

Hamiltonian: General momentum $p \equiv \frac{\partial L}{\partial \dot{q}}$. Introduce the \`Hamiltonian'. $H(q,p) = p\dot{q} - L = T + U$,

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The equations of motion are called Hamilton's equations

$$\dot{q} = \frac{\partial H}{\partial p} \qquad \dot{p} = -\frac{\partial H}{\partial q}.$$

q and p are independent variables! 1st order ODEs, general coords.

Initial data are $(q(0), p(0))$ Again we have no constraint like ~~$\dot{q} = p$.~~

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Example: Harmonic oscillator with mass $m = 1$

$$T = \frac{p^2}{2m} = \frac{p^2}{2},\ U = \frac{1}{2}kq^2 = \frac{1}{2}m\omega^2 q^2 = \frac{1}{2}\omega^2 q^2,$$

$$H(q,p) = T + U = \frac{1}{2}(p^2 + \omega^2 q^2).$$

Let's check

$$\frac{dH}{dt} = \frac{\partial H}{\partial q}\frac{dq}{dt} + \frac{\partial H}{\partial p}\frac{dp}{dt} = \omega^2 q\dot{q} + p\dot{p} = -\dot{p}\dot{q} + \dot{q}\dot{p} = 0.$$

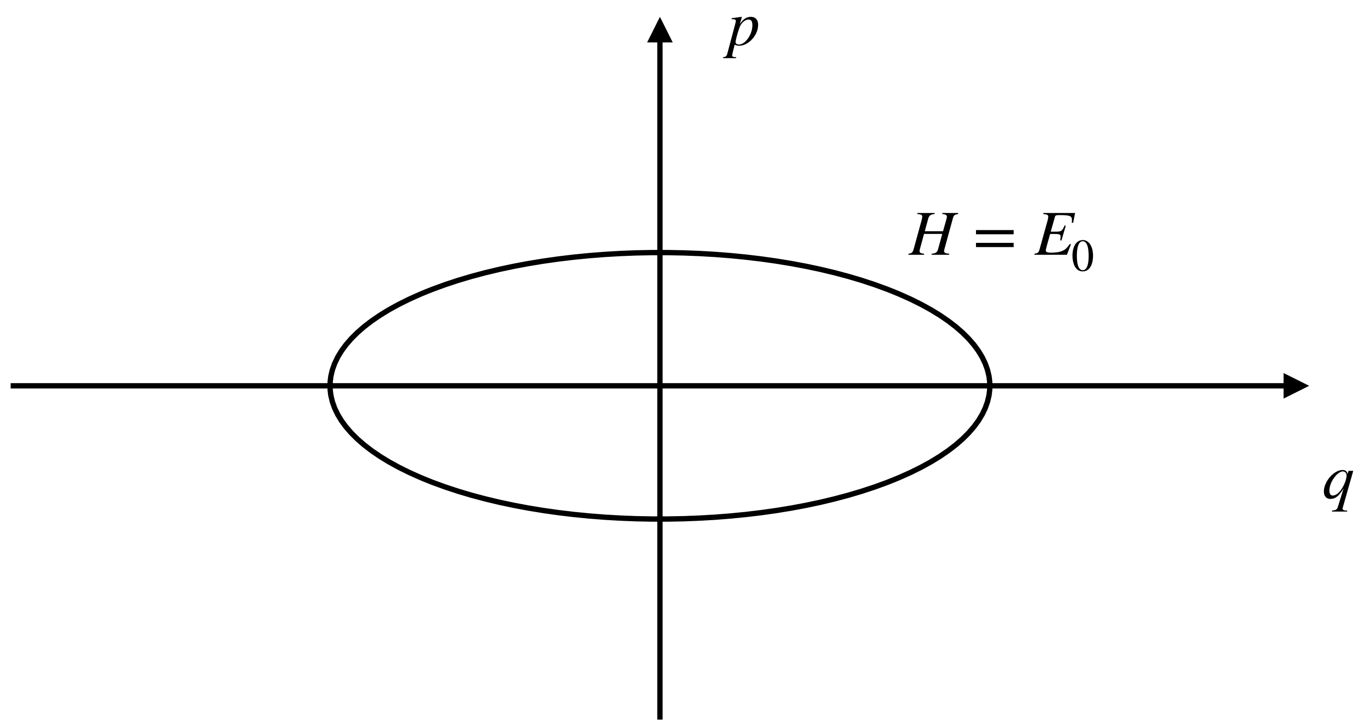
So, we've just proved conservation of energy in great generality:

$$H = E.$$

III. Classical Mechanics

Proved conservation of energy in great generality:

$$H = \frac{1}{2}(\omega^2 q^2 + p^2) = E.$$



By purely geometrical means we've picked out the orbit of the harmonic oscillator. Preview: we'll use these tools first to picture what quantum mechanics tells us about the allowed orbits and then to come to a geometrical understanding of entropy.