Today

I. Last Time
II. Classical Mechanics, Quantum Mechanics, and Entropy
III. Probabilities and Physics

I. Reviewed Mechanics: Lagrangian mechanics works in any coordinates whatsoever. In Hamiltonian mechanics the equations of motion become 1st order ODEs

$$\dot{q} = \frac{\partial H}{\partial p} \qquad \text{and} \qquad \dot{p} = -\frac{\partial H}{\partial q}.$$

We also found that these equations led to conservation of energy (whenever H is time independent).

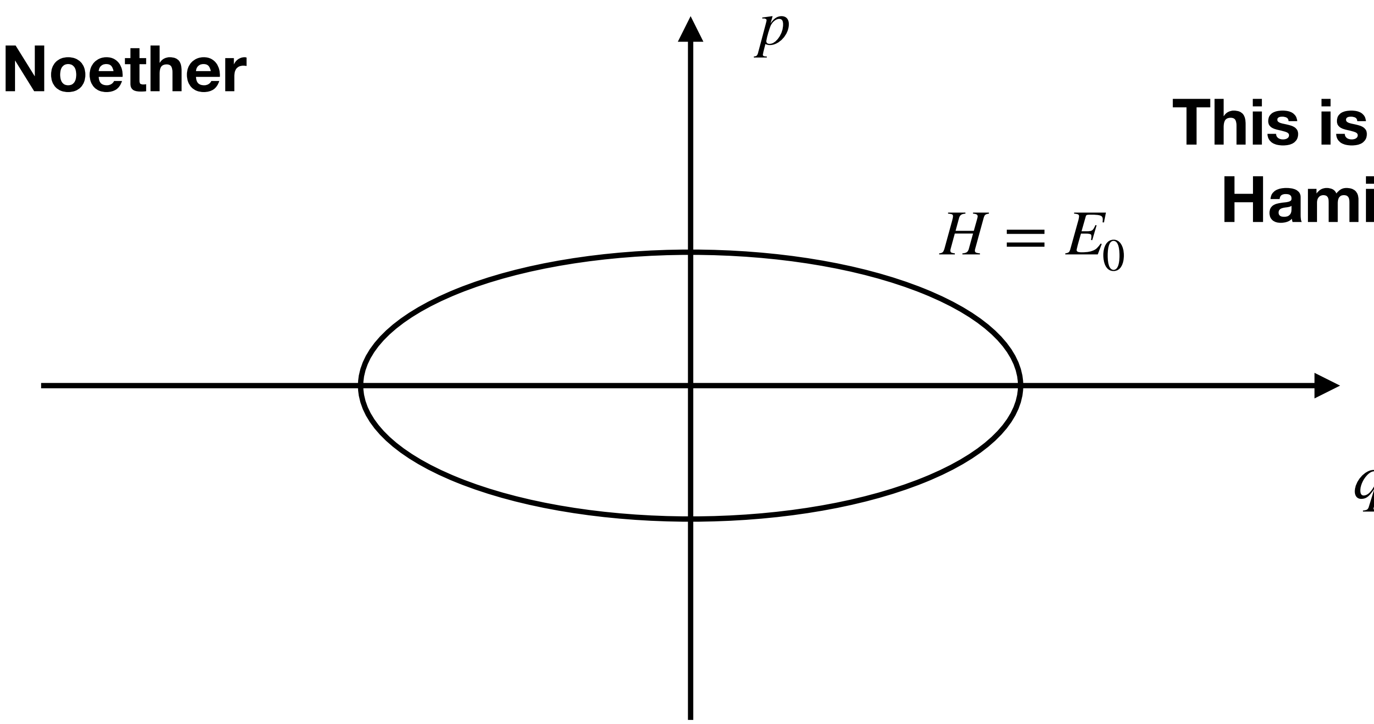
The first order nature of these equations means that trajectories in phase space (q, p) never cross.

I. <u>Classical Mechanics</u>

Proved conservation of energy in great generality:

$$H = \frac{1}{2}(\omega^2 q^2 + p^2) = E.$$

Amelie (Emmy) Noether

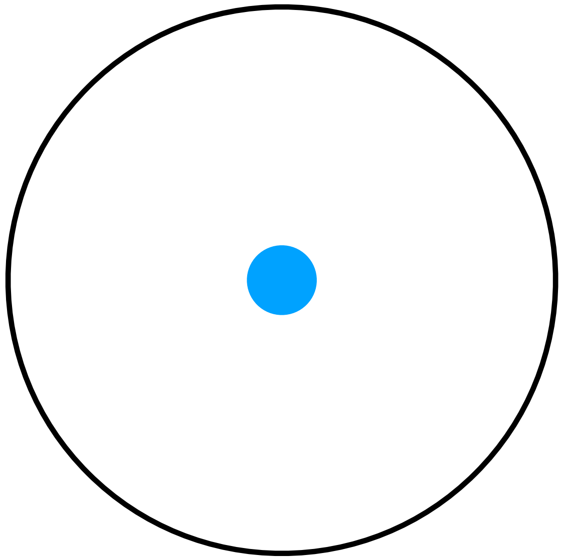


This is a `level set' of the Hamiltonian function.

This last formulation of mechanics, Hamilton's, is particularly useful to us because of its relations to Quantum Mechanics. I want to explain the "Old Quantum Theory"

II. Old Quantum Theory

This began with Bohr's description of the atom. In thinkin about the electron as a sort of matter wave, Bohr realized that it should be subject to boundary conditions.



This was accompanied by a remarkable discovery that angular momentum should take on a discrete set of values:

$$L = n\hbar \quad n = 1,2,3,\cdots.$$

Here $\hbar$ is Planck's constant $\hbar = 1.05 \times 10^{-34} m^2 kg/s$

II. Old Quantum Theory

Physical Input 1: Observables (like L) that appeared to be continuous classically can be discrete, taking only certain values, in quantum mechanics.

Which observables? A hunt began for which observables and why.

In a 2D phase space the result is easy to state. Consider a classical observable $A = A(q, p)$ (e.g. energy), if the level set $A = a$ (where a is some value) captures a finite area in the phase space, then the observable A is quantized.

II. Old Quantum Theory

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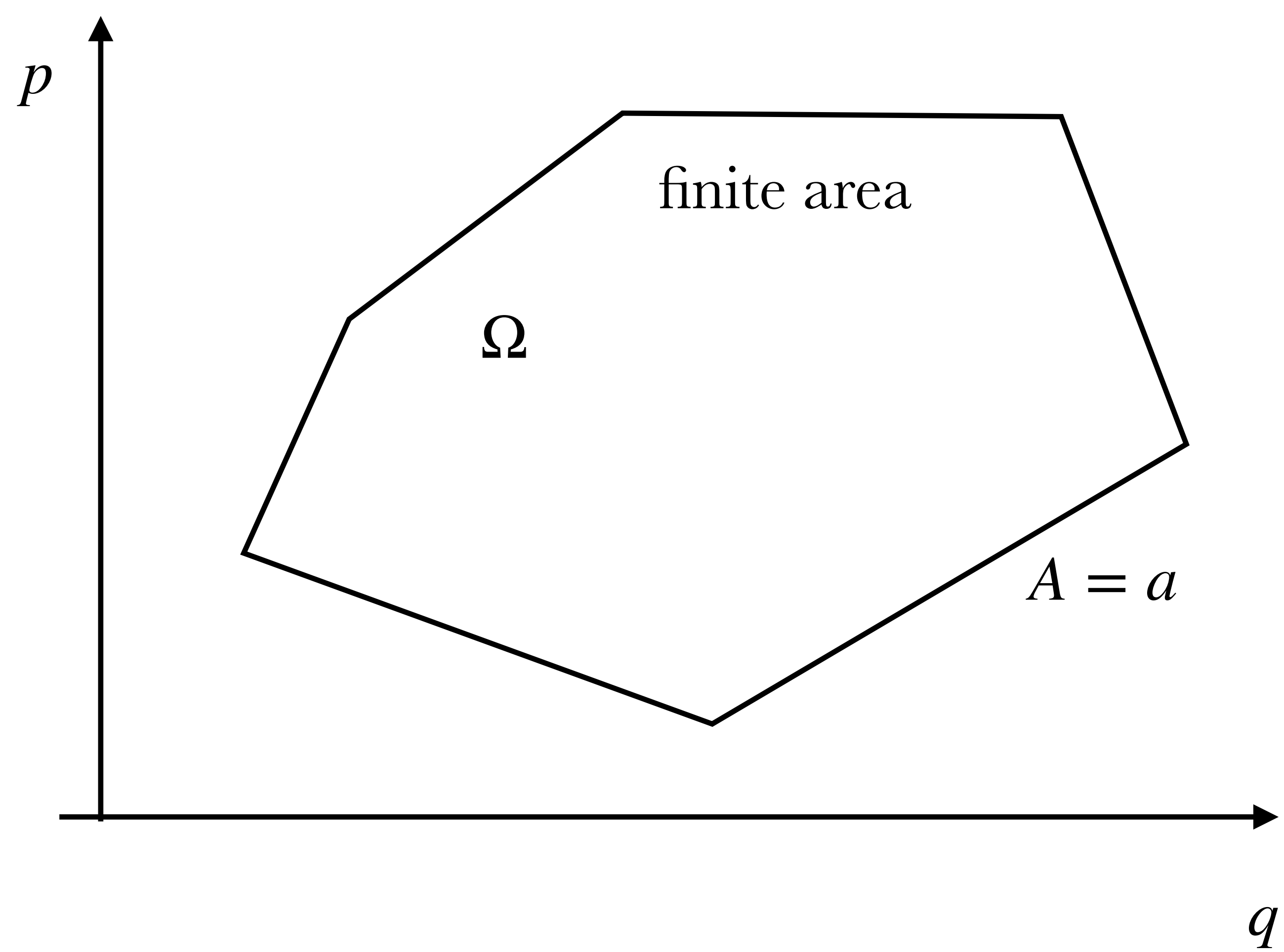


They even discovered an approximation for the quantized values of the observable. The area inside $A = a$ is

$$Area = \int_{\Omega} dqdp$$
$$= \oint_{\partial\Omega} pdq$$

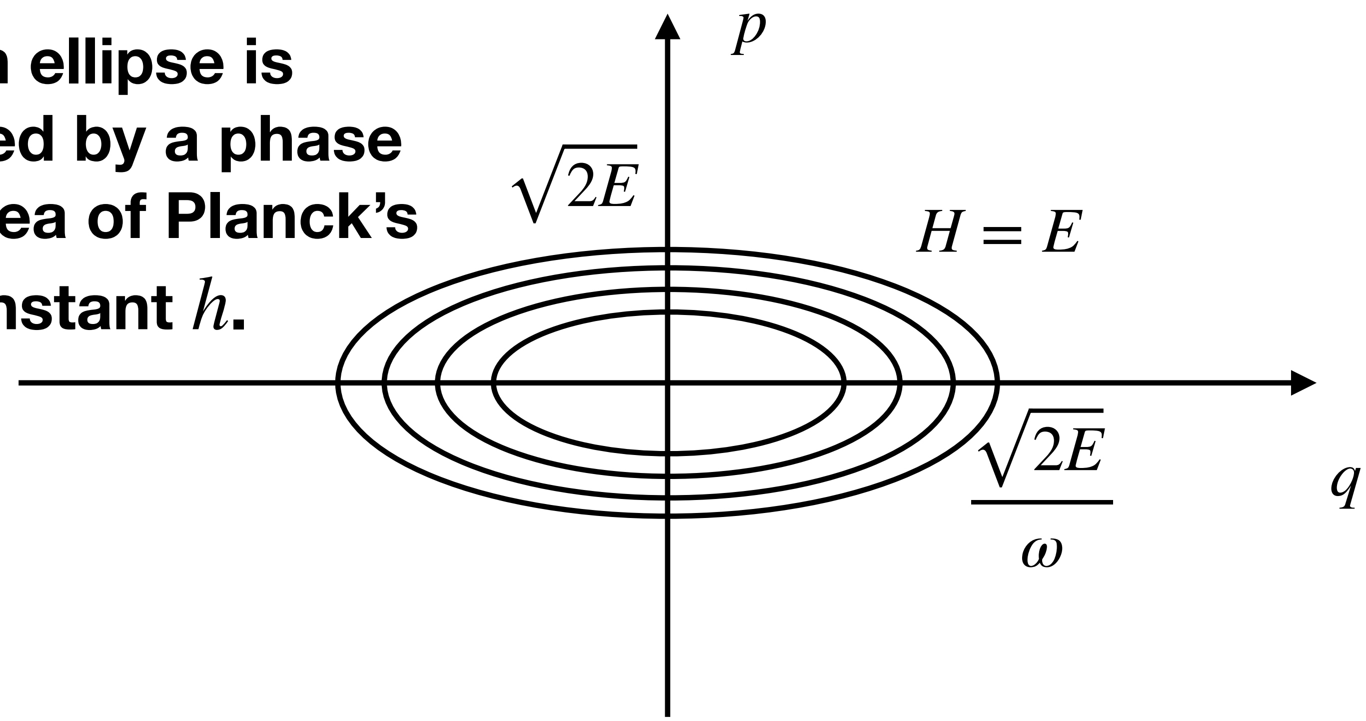
The approximation that they was

$$S(A) \equiv \oint pdq = (n + \frac{1}{2})h = 2\pi(n + \frac{1}{2})\hbar.$$

II. Old Quantum Theory

Let's try it on our harmonic oscillator: $H = \frac{1}{2}(\omega^2 q^2 + p^2) = E$

Each ellipse is separated by a phase space area of Planck's constant h.



The area of an ellipse is $Area = \pi\sqrt{2E}\frac{\sqrt{2E}}{\omega} = \pi\frac{2E}{\omega}$. Then,

$$\frac{2\pi E}{\omega} = 2\pi(n + \frac{1}{2})\hbar \implies E_n = (n + \frac{1}{2})\hbar\omega.$$

The energy of the harmonic oscillator is quantized!

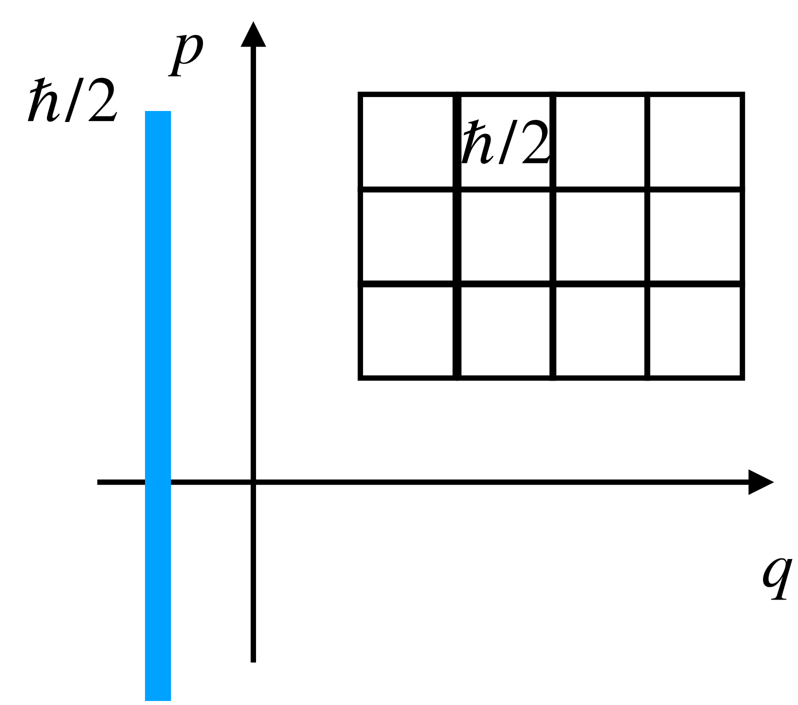
II. Old Quantum Theory

Physical input 2 (uncertainty principle): Measurement of a system inevitably disrupts its state; so you can't know the position and the momentum of a particle to arbitrary precision:

$$\Delta q \Delta p \geq \frac{\hbar}{2}.$$

It turns out that there are physical states that saturate this bound (so-called coherent states). These states divide the phase space up into Planck sized cells too.

Thus in the Quantum picture we should think of the phase space as being made up of tiny Planck sized cells, not of points.



For us, this is the most important thing to learn from Quantum Theory.

II. Old Quantum Theory

Physical input 3: Nature is intrinsically probabilistic. Describe quantum states by smooth functions $\psi(q)$ and expected outcomes of measurements, say $\hat{A}$, by

$$\langle \hat{A} \rangle = \int_{-\infty}^{\infty} \psi^*(q) \hat{A} \psi(q) dq.$$

(Don't need to know this, it's only here for completeness.)

III. How is all of this useful to us?

We can answer this through the terminology of microstates and macrostates. This quantum foundation is going to allow us to think about multiplicities.