I.   Reviewed Mechanics: Lagrangian mechanics works in any coordinates whatsoever. In Hamiltonian mechanics the equations of motion become 1st order ODEs

$$\dot{q} = \frac{\partial H}{\partial p} \quad \text{and} \quad \dot{p} = -\frac{\partial H}{\partial q}.$$

We also found that these equations led to conservation of energy (whenever H is time independent).

The first order nature of these equations means that trajectories in phase space (q, p) never cross.

I. Old Quantum Theory gives us a way to find the quantized energy of any particular observable $A = A(q, p)$ if captures finite area in phase space:

$$\int_\Omega dp\, dq = \oint p\, dq = (n + \frac{1}{2})h = 2\pi(n + \frac{1}{2})\hbar.$$

This is a general property of quantum states. The uncertainty principle tells us that we cannot localize quantum states better than to a Planck cell in the phase space.
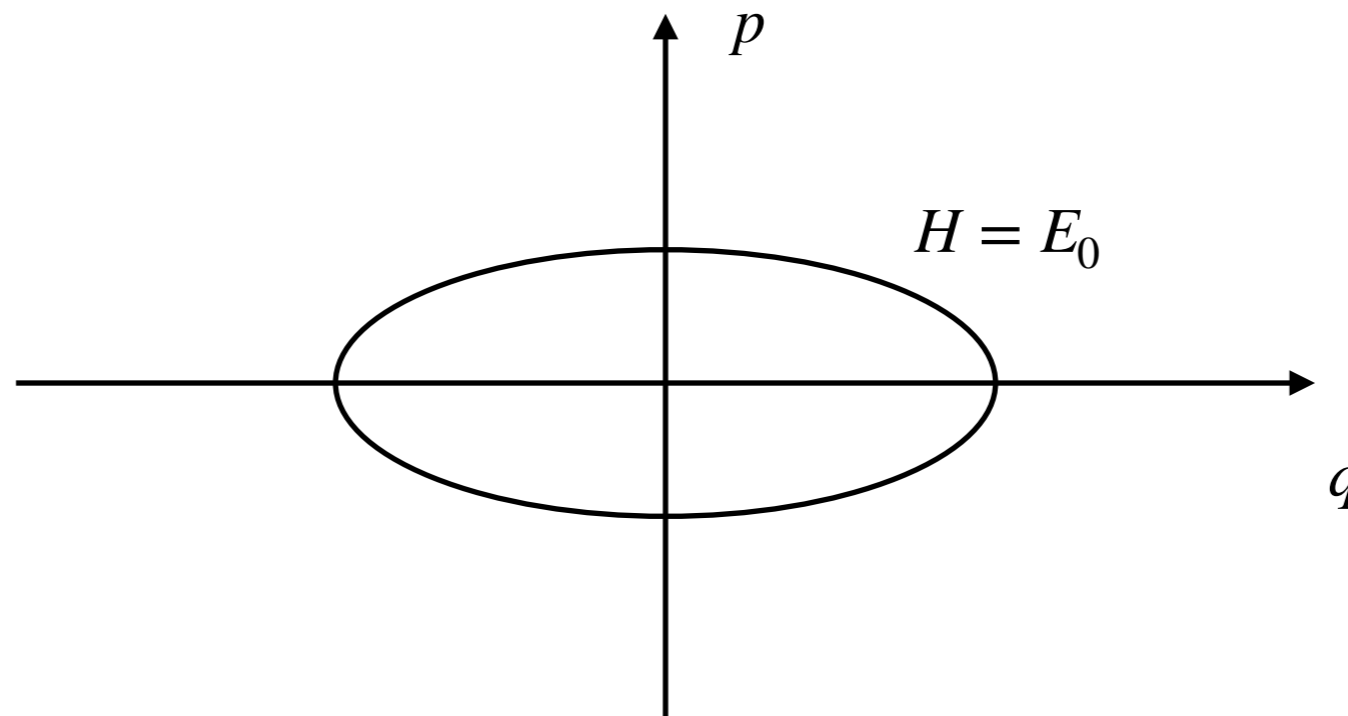
Quantum Mechanics breaks phase space up into Planck sized cells.

I. Quantum Mechanics breaks phase space up into Planck sized cells.

II. Fundamental Postulate of Statistical Mechanics: An isolated system in equilibrium is equally likely to be in any of its accessible states.

Here accessible is doing the heavy lifting of telling us that we have macroscopic constraints on our system. The accessible states are those that satisfy the constrains.
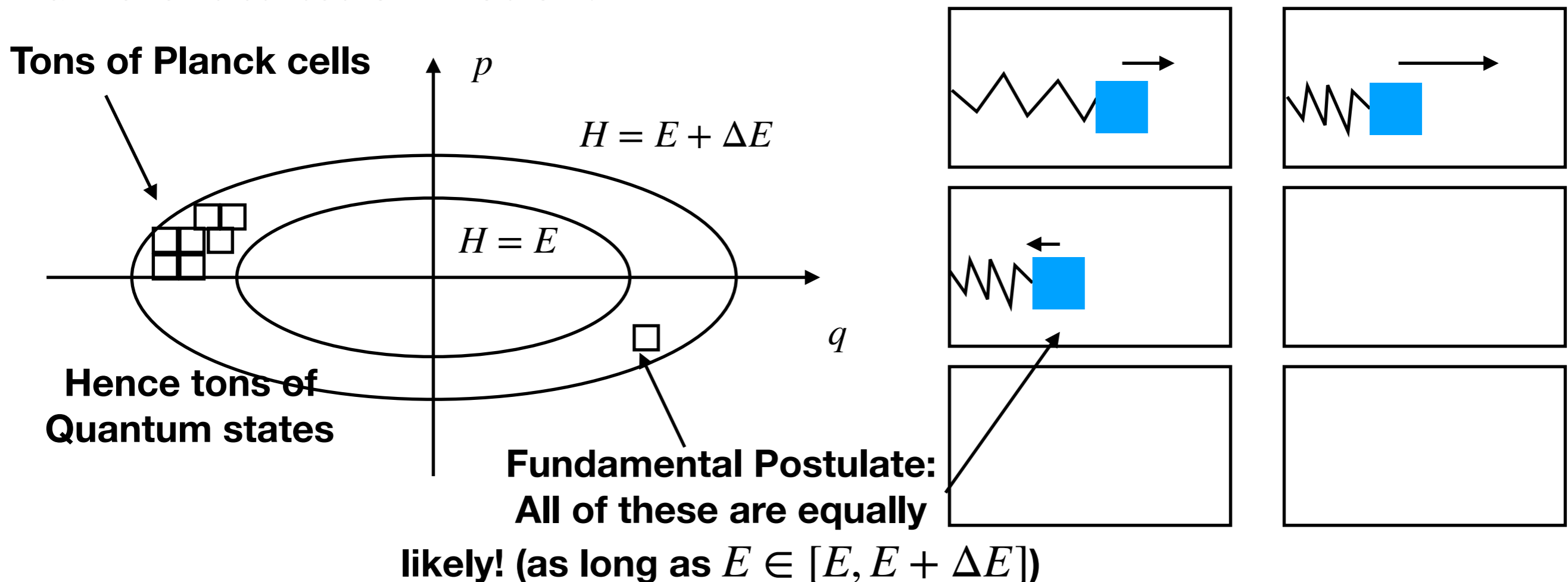
$$H = E + \Delta E$$

$$H = E$$
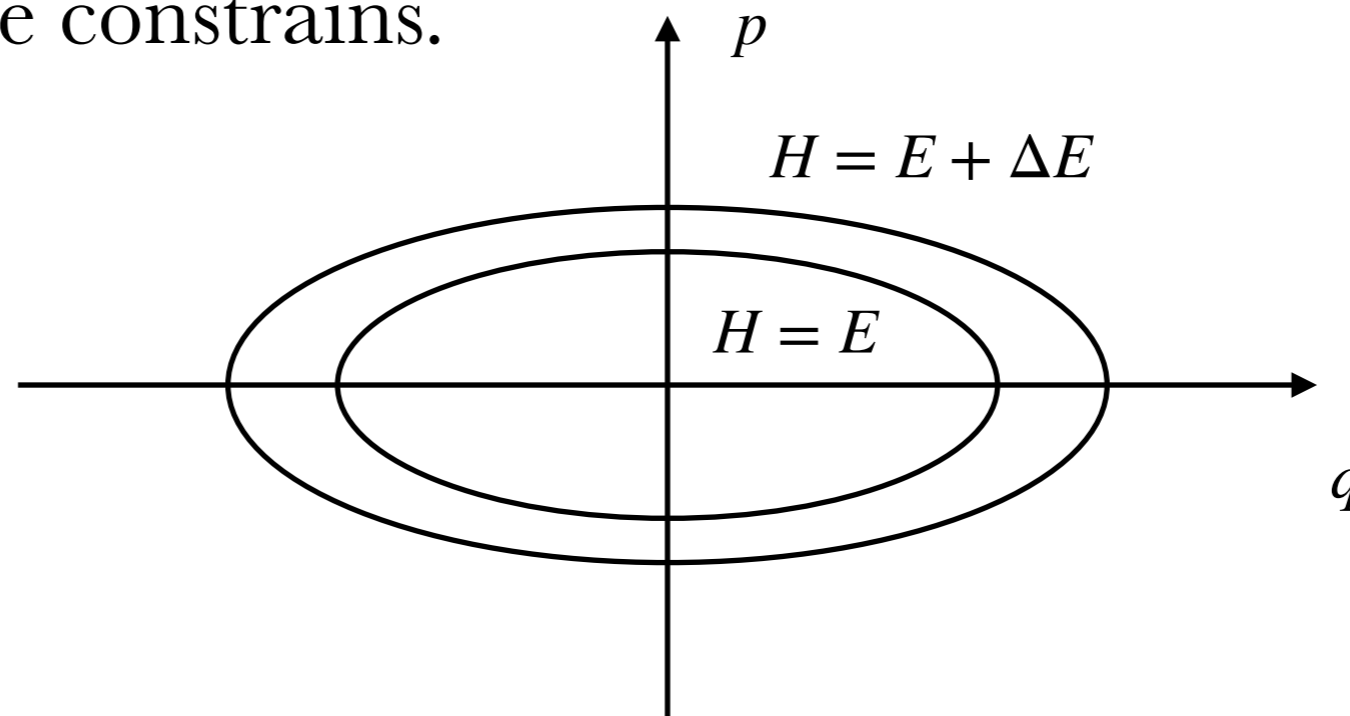
In this example, we've constrained the energy to lie between E and $E + \Delta E$.

II. <u>Ex.</u> Our oscillator again (take $m = 1$)

$$H = \frac{1}{2}(\omega^2 q^2 + p^2).$$

Measured to have an energy in the interval $[E, E + \Delta E]$.

To talk about this we think of an <u>ensemble</u>: pictured at right.. Many instances of the oscillator, all with energy in $[E, E + \Delta E]$, but having different states of motion.

Tons of Planck cells

p

$H = E + \Delta E$

$H = E$

q

Hence tons of Quantum states

Fundamental Postulate: All of these are equally likely! (as long as $E \in [E, E + \Delta E]$)

II. Probabilities and Physics:

<u>Def:</u> $\Omega(E)$ is total number of states with energy in $[E, E + \Delta E]$. (Microcanonical Ensemble). We'll often approximate the Range either by $[0,E]$ or by infinitely precise and just E.

Similarly, let $\Omega(E, y)$ be the number of states with energy in $[E, E + \Delta E]$ *and* with $y = y_0$. You've seen an example of this with $E = q$ units of energy and with $y = N$ oscillators, the Einstein solid.

What's the probability of the macroscopic y_0?

$$P(y_0) = \frac{\Omega(E, y_0)}{\Omega(E)}.$$

"All states are equally likely, so probability of finding y_0 is just the fraction of states that have $y = y_0$."

III. Return of the Ideal Gas

An ideal gas is a collection of a large number of molecules or atoms held in a box of volume V and with some total energy U.

Let's consider one gas molecule first. This molecule can be anywhere in the box, so it has a spatially allowed volume of V. The allowed volume in momentum space will take some work to pin down, so for the moment let's just call it V_p. Then phase space volume is

$$\Omega_1 \propto V \cdot V_p.$$

To get the momentum space volume V_p we're going to need to understand the energy constraint. What is our constraint explicitly in terms of the momenta?

$$U = \frac{1}{2}m(v_x^2 + v_y^2 + v_z^2) = \frac{1}{2m}(p_x^2 + p_y^2 + p_z^2)$$

III. Return of the Ideal Gas

To get the momentum space volume V_p we're going to need to understand the energy constraint. What is our constraint explicitly in terms of the momenta?

$$U = \frac{1}{2}m(v_x^2 + v_y^2 + v_z^2) = \frac{1}{2m}(p_x^2 + p_y^2 + p_z^2)$$

This is a sphere of radius $\sqrt{2mU}$ in momentum space. Hence the allowed surface area is: $8\pi mU$.

We'd also like to understand the proportionality constant above: Both units and the analysis that we've done over the last week tell us that we have to divide by the volume in phase space of a single quantum state, which is h^3. Hence we get

$$\boxed{\Omega_1 = \frac{V \cdot V_p}{h^3}.}$$