Today

I. Last Time
II. Distinguishable Particles, Bosons, and Fermions
III. Identical Particle Distribution Functions

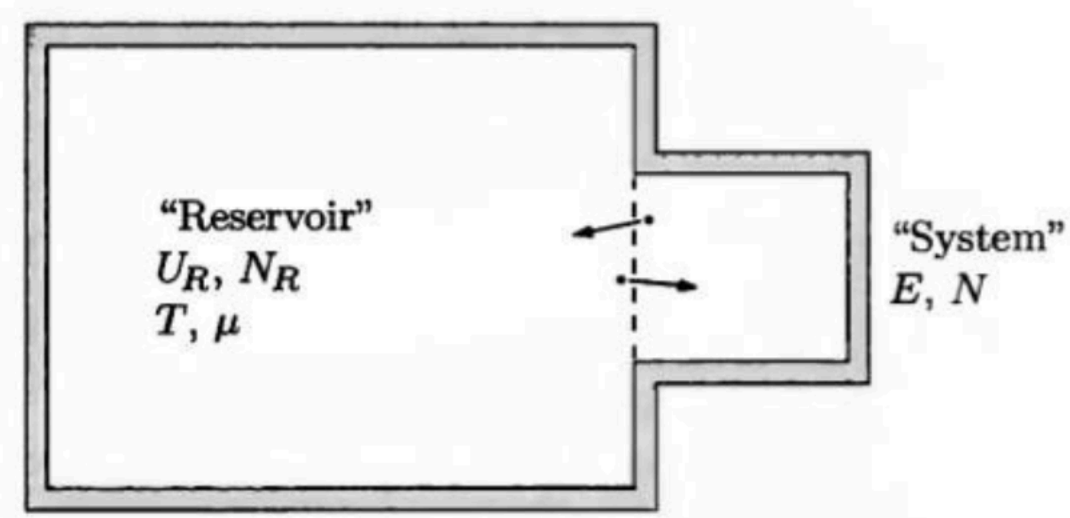
I. Spencer gave a guest lecture on Gibbs factors and the grand partition function

$$e^{-(E(s)-\mu N(s))/kT}.$$

The grand partition function is sum over these Gibbs factors

$$\mathcal{Z} = \sum_s e^{-(E(s)-\mu N(s))/kT}.$$

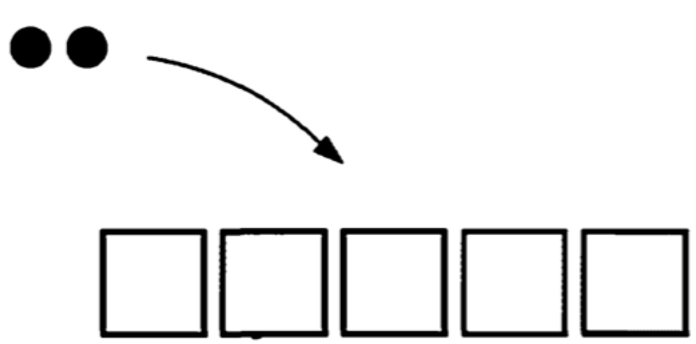
The setup for these is a system in contact with a constant temperature reservoir that can also exchange particles.



II. Let's return to our discussion of identical particles and their indistinguishability. Previously we had argued that we could compensate for the over counting of the distinguishable particle case by dividing by $N!$, and got

$$Z = \frac{1}{N!}(Z_1)^N.$$

This doesn't work for quantum particles. To show that we take a system of five states and two particles, but with all the states having 0 energy.



11000	01010	20000
10100	01001	02000
10010	00110	00200
10001	00101	00020
01100	00011	00002

$Z = 5 \times 5 = 25$ (distinguishable) $Z = \frac{1}{2}25 = 12.5$ (Indistinguishable)

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How do we deal with this in general?

The assumption we've made to get the counting $Z = 15$ is that the particles are **bosons**, that is, they can occupy exactly the same state.

On the other hand **fermions** are particles such path only one can occupy any state at a time.

A remarkable theorem of relativistic quantum mechanics is that the spin of a particle tells you its statistics (i.e. whether it is a boson or a fermion). For example,

Bosons: photons, gluons, Higgs boson … (spins=0,1,2)

Fermions: electrons, neutrons, neutrinos, protons, (spins=1/2, 3/2,5/2,…)

These ideas lead us to distinguish boson and fermionic partition func

II. Why have we been getting away with what we've been doing for so long?

It turns out that we've been focused on low density systems. That is,

$$Z_1 \gg N.$$

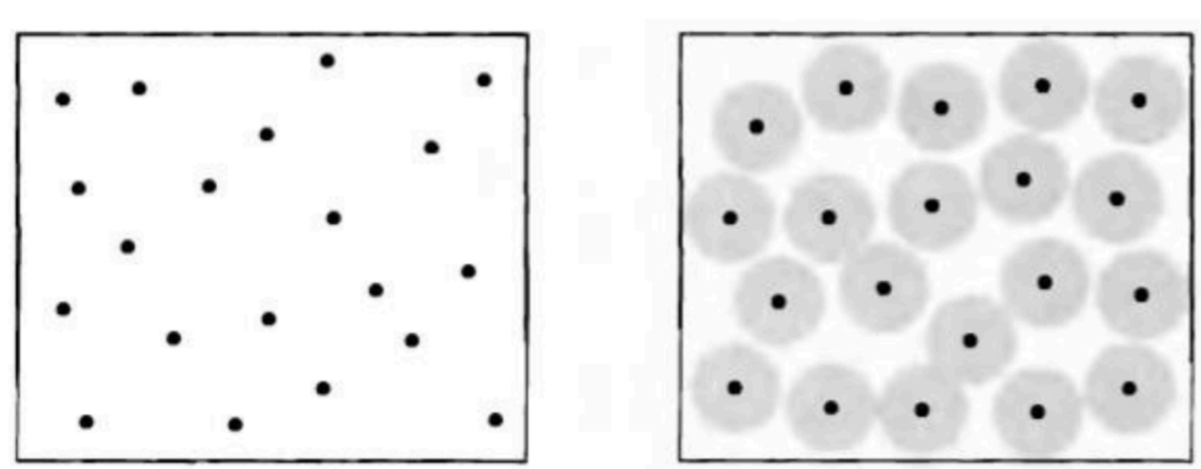
More precisely, recalling

$$v_Q = \ell_Q^3 = \left(\frac{h}{\sqrt{2\pi mkT}} \right)^3,$$

the condition can be stated as

$$\frac{V}{N} \gg v_q.$$

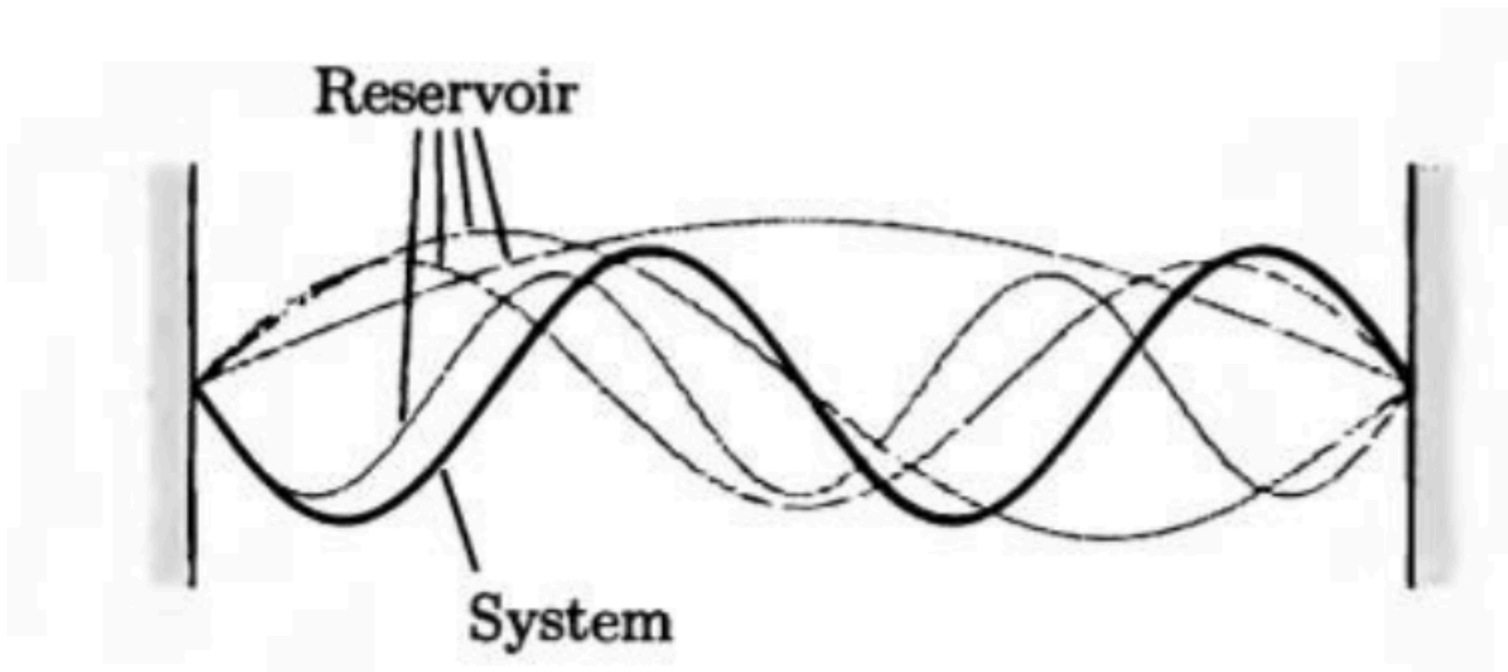
Let's drop this assumption.



Normal gas, $V/N \gg v_Q$ Quantum gas, $V/N \approx v_Q$

III. Let's turn to distribution functions.

The brilliant idea here is to treat a single energy level as our system and all the remaining energy levels as our reservoir.



Suppose, in general, that there are n particles in our state of energy ϵ and chemical potential μ. Then

$$P(n) = \frac{1}{\mathcal{Z}} e^{-(n\epsilon - \mu n)/kT} = \frac{1}{\mathcal{Z}} e^{-n(\epsilon - \mu)/kT}.$$

Let's study these probabilities for fermions.

III. Then

$$P(n) = \frac{1}{\mathscr{Z}} e^{-(n\epsilon - \mu n)/kT} = \frac{1}{\mathscr{Z}} e^{-n(\epsilon - \mu)/kT}.$$

Let's study these probabilities for fermions. Only one or zero fermions is allowed to occupy our single state and so we have

$$\mathscr{Z} = 1 + e^{-(\epsilon - \mu)/kT}.$$

Then, the average number of particles is

$$\bar{n} = \sum_n nP(n) = \frac{e^{-(\epsilon-\mu)/kT}}{1 + e^{-(\epsilon-\mu)/kT}} = \frac{1}{e^{(\epsilon-\mu)/kT} + 1}.$$

This is so widely used that it has its own name, the Fermi-Dirac,

$$\bar{n}_{FD} = \frac{1}{e^{(\epsilon-\mu)/kT} + 1}$$

Let's plot this and try to understand its shape as a function of energy.

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