Today

I. Last Time
II. Degenerate Fermi Gas
III. Saiqi's Guest Lecture on Black Body Radiation

I. Last time we derived the occupancy of a Bose-Einstein gas

$$\bar{n}_{BE} = \frac{1}{e^{(\epsilon-\mu)/kT} - 1}.$$

Fermi energy: $\epsilon_F \equiv \mu(T=0)$.

Let's again fix our attention on a cubical box of side length L (we're think of it as a chunk of metal). Recall that free particles have $\lambda_n = \frac{2L}{n}$, and $p_n = \frac{h}{\lambda_n} = \frac{hn}{2L}$, but this time let's take into account the 3D nature of the box $p_x = \frac{hn_x}{2L}$, $p_y = \frac{hn_y}{2L}$, $p_z = \frac{hn_z}{2L}$. The energies

$$\epsilon(\overrightarrow{n}) = \frac{|\overrightarrow{p}|^2}{2m} = \frac{h^2}{8mL^2}(n_x^2 + n_y^2 + n_z^2).$$

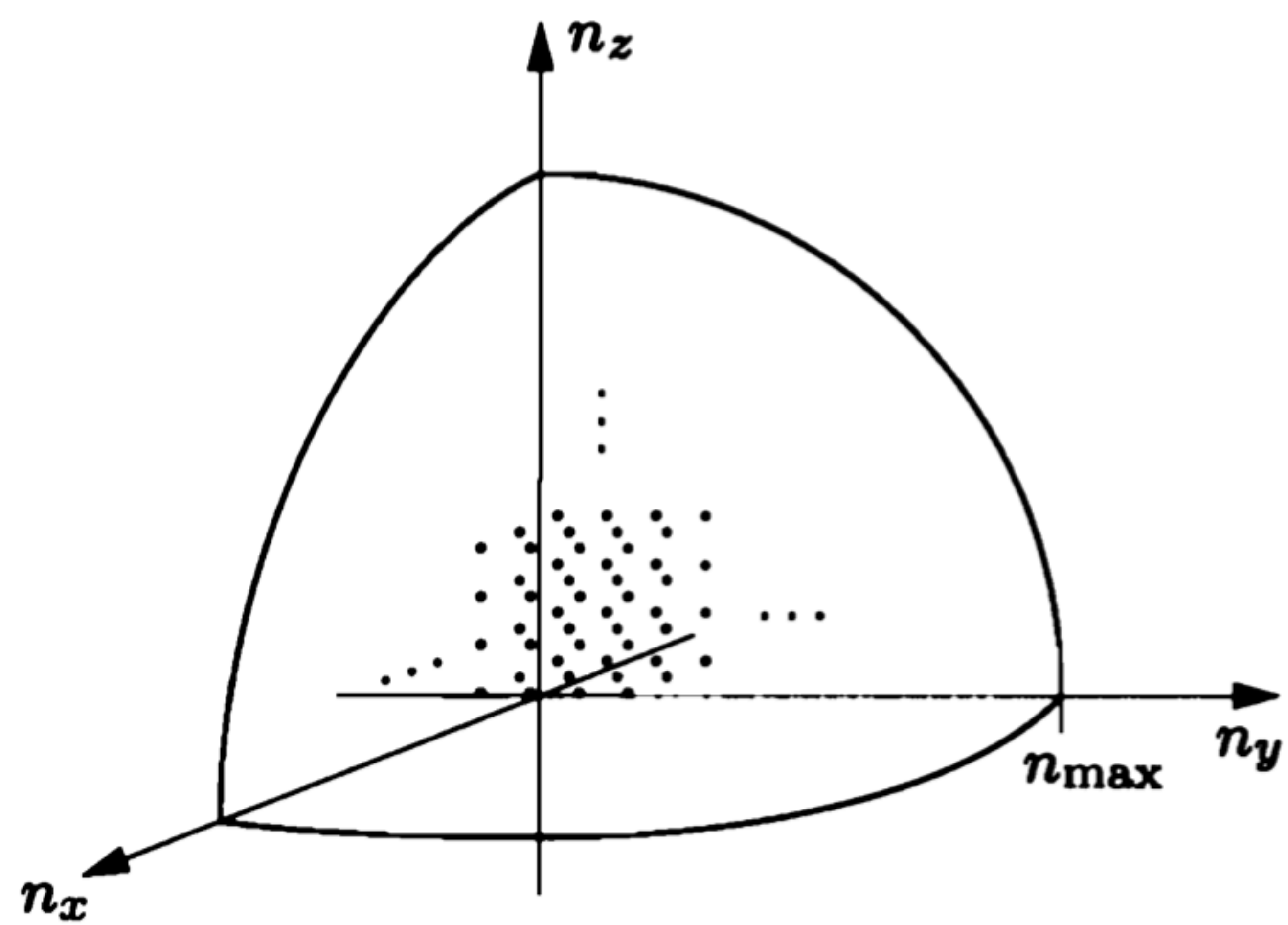
II. We'd like to compute the total number of allowed quantum states and to do that we have to count the lattice sites for positive integer n_x, n_y, n_z and a very convenient way to do that is to compute the volume of the octant of a sphere in n-space. Then we have

$$N = 2 \times (\text{vol of octant of sphere}) = 2 \cdot \frac{1}{8} \cdot \frac{4}{3}\pi n_{\text{max}}^3 \,,$$

here $n = \sqrt{n_x^2 + n_y^2 + n_z^2}$.

On the other hand, we can solve the above for n_{max} in terms of N, and plug in to energy to get

$$\epsilon_F = \frac{h^2 n_{\text{max}}^2}{8mL^2} = \frac{h^2}{8m}\left(\frac{3N}{\pi V}\right)^{2/3}$$



II. Now that we know how far to go we can compute the U:

$$U = 2\sum_{n_x}\sum_{n_y}\sum_{n_z}\epsilon(\overrightarrow{n}) \approx 2\int_0^{n_{max}}\int_0^{\pi/2}\int_0^{\pi/2} n^2 \sin\theta\epsilon(n)d\phi d\theta dn = \pi\int_0^{n_{max}} \epsilon(n)n^2 dn$$

$$= \pi\int_0^{n_{max}} \frac{h^2n^2}{8mL^2}n^2 dn = \frac{\pi h^2}{40mL^2}n_{max}^5 = \frac{3}{5}N\epsilon_F$$