Today

I. Announcements: No class Wednesday (advising days), One final class on Monday of completion days (Dec. 14th), Final due Friday Dec. 18th at 10pm ET
II. Last Time
III. Physical Scales for Degenerate Fermi Gas
IV. Density of States
V. Bruno's Guest Lecture on Bose-Einstein Condensation

I. Last time Saiqi gave a guest lecture on black body radiation. Saiqi introduced the notion that some bosons , e.g. photons, have zero chemical potential and this was essential to the black body results. Saiqi showed us Planck's derivation of the spectral energy density $u(T,\lambda) = \dfrac{U(T,\lambda)}{V}$.

Working in the $T = 0$ we found $U = \dfrac{3}{5}N\epsilon_F$, $\epsilon_F = \dfrac{h^2}{8m}\left(\dfrac{3N}{\pi V}\right)^{2/3}$.

I. If you put in values for all the constants in the Fermi energy and reasonable numbers for $N$ and $V$, you will find

$\epsilon_F \sim 2 - 3\text{eV}$, $kT \sim \dfrac{1}{40}$ eV, so comparing these gives

$kT \ll \epsilon_F$, it's a quick check that this is the same as $\dfrac{V}{N} \ll v_Q$.

Folks often speak about the Fermi temperature, which is defined

$$T_F \equiv \frac{\epsilon_F}{k}.$$

(Not too useful in practice, because it is well above the melting point of most metals.) The limit in which $T \ll T_F$ is the defining limit ofr a degenerate Fermi gas. We can also find the **degeneracy pressure**

$$P = -\left(\frac{\partial U}{\partial V}\right)_{S,N} = -\frac{\partial}{\partial V}\left[\frac{3}{5}N\frac{h^2}{8m}\left(\frac{3N}{\pi}\right)^{2/3} V^{-2/3}\right] = \frac{2N\epsilon_F}{5V} = \frac{2}{3}\frac{U}{V}$$

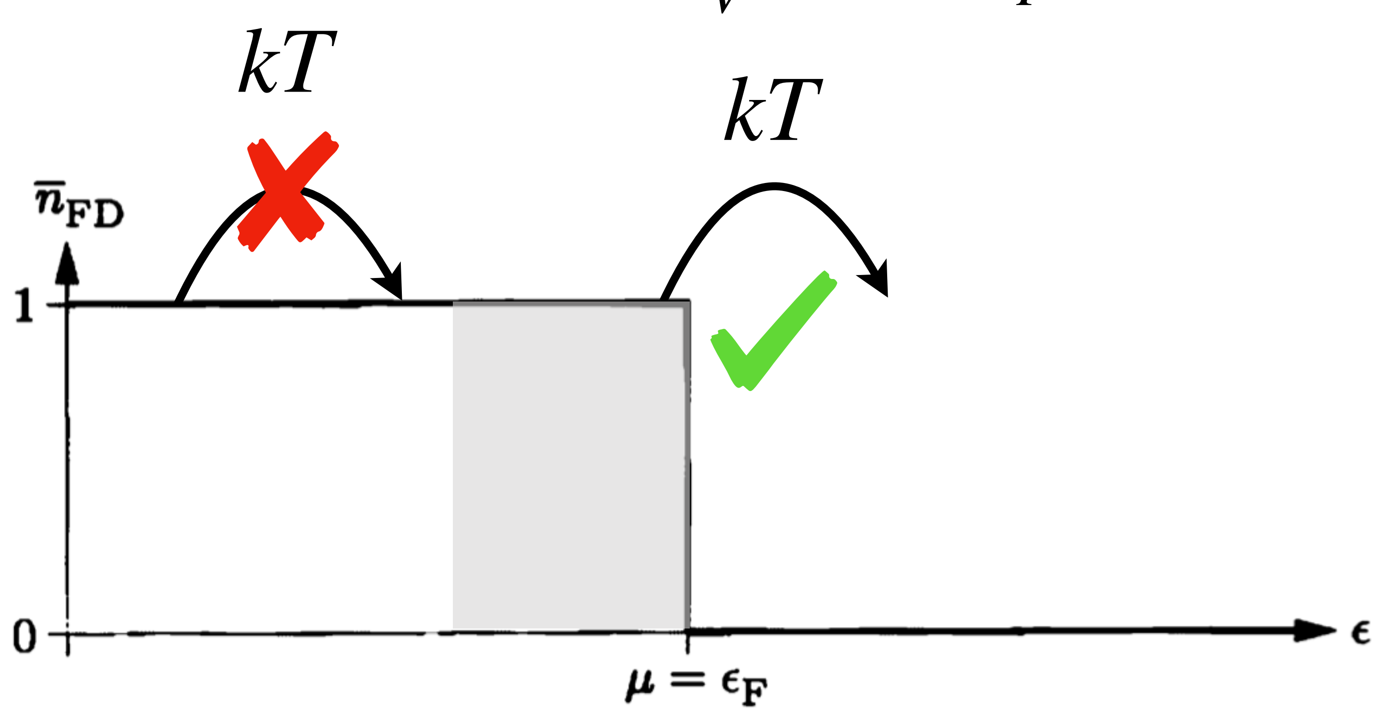
II. Let's move away from the zero temperature limit and consider small temperatures. What additional energy enters the system due to the thermal excitation of some (but not all) of the electrons?

additional energy $\propto$ (number of particles affected)(energy acquired by constituents) $\propto (kTN)(kT)$.

The full result: $U = \frac{3}{5}N\epsilon_F + \frac{\pi^2}{4}N\frac{(kT)^2}{\epsilon_F}$.

This gives a heat capacity in the usual way:

$$C_V = \left.\frac{\partial U}{\partial T}\right|_V = \frac{\pi^2 N k^2 T}{2\epsilon_F}.$$



III. Density of states.

Let's recast our computation of the energy of the Fermi gas in terms of the energies of the states:

$$\epsilon = \frac{h^2}{8mL^2}n^2,\ n = \sqrt{\frac{8mL^2}{h^2}}\sqrt{\epsilon},\ \text{then } dn = \sqrt{\frac{8mL^2}{h^2}}\frac{1}{2\sqrt{\epsilon}}d\epsilon$$

$$U = \pi\int_0^{n_{max}} \frac{h^2n^2}{8mL^2}n^2 dn = \int_0^{\epsilon_F} \epsilon \left[\frac{\pi}{2}\left(\frac{8mL^2}{h^2}\right)^{3/2}\sqrt{\epsilon}\right] d\epsilon$$