Today

I. Announcements: One final class on Monday of completion days (Dec. 14th), Final due Friday Dec. 18th at 10pm ET
II. Last Time
III. Density of States Wrap Up
IV. Balthazar's Guest Lecture on Debye Theory of Solids
V. CAFE forms for Thermal: https://tools.bard.edu/tools/cafeform/

I. Bruno told us about Bose-Einstein condensation. At low temperatures all Bosons drop into the lowest energy state and they behave collectively in quantum mechanical way.

Introduced the idea of the density of states: divide energy space into bands, then the density of states tells me how many quantum states there are in a given band of energies.

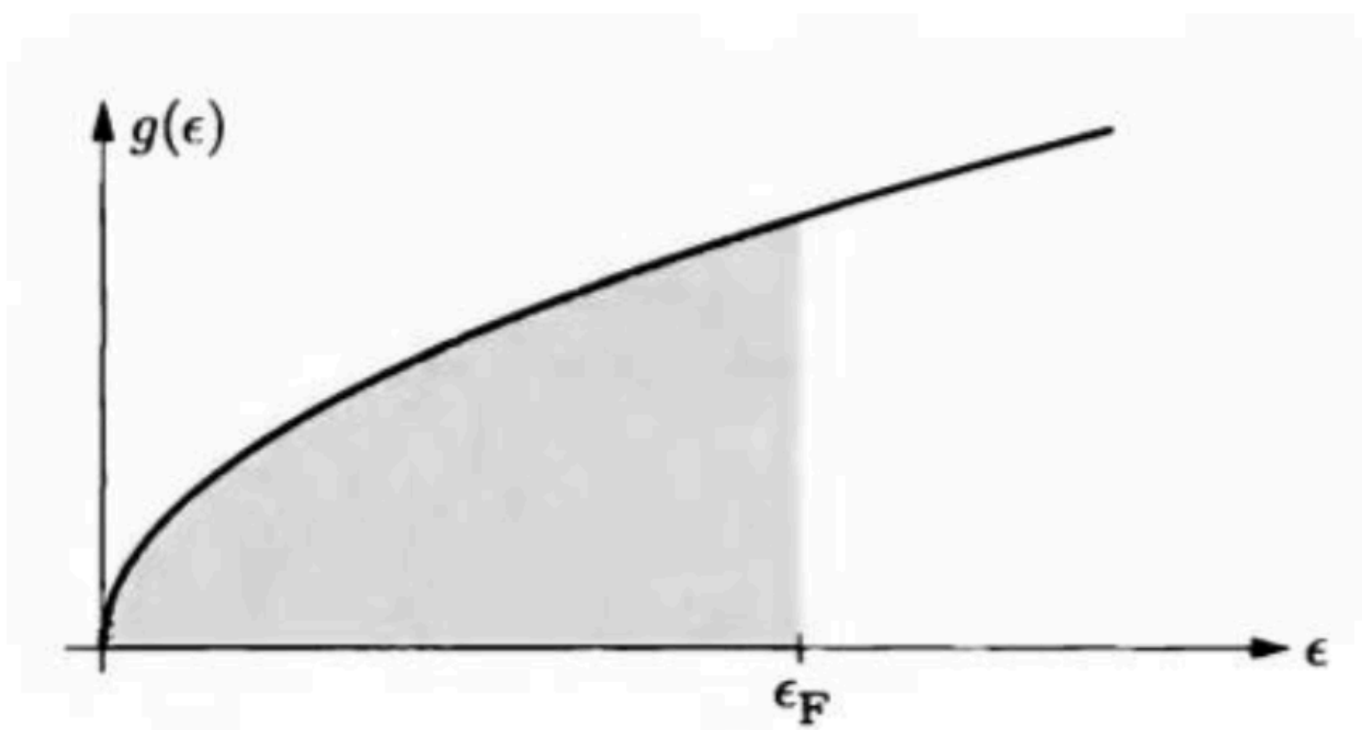
II. Density of states.

Let's recast our computation of the energy of the Fermi gas in terms of the energies of the states:

$$\epsilon = \frac{h^2}{8mL^2} n^2, \text{ gives } n = \sqrt{\frac{8mL^2}{h^2}} \sqrt{\epsilon}, \text{ and so } dn = \sqrt{\frac{8mL^2}{h^2}} \frac{1}{2\sqrt{\epsilon}} d\epsilon$$

$$U = \pi \int_0^{n_{max}} \frac{h^2 n^2}{8mL^2} n^2 dn = \int_0^{\epsilon_F} \epsilon \left[\frac{\pi}{2} \left(\frac{8mL^2}{h^2} \right)^{3/2} \sqrt{\epsilon} \right] d\epsilon$$

$g(\epsilon)$



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Note that we can write this in one more way:

$$g(\epsilon) = \frac{\pi(8m)^{3/2}}{2h^3}V\sqrt{\epsilon}.$$

By definition we can also compute the number of particles in our system using this: $N = \int_0^{\epsilon_F} g(\epsilon)d\epsilon \quad (T=0)$.

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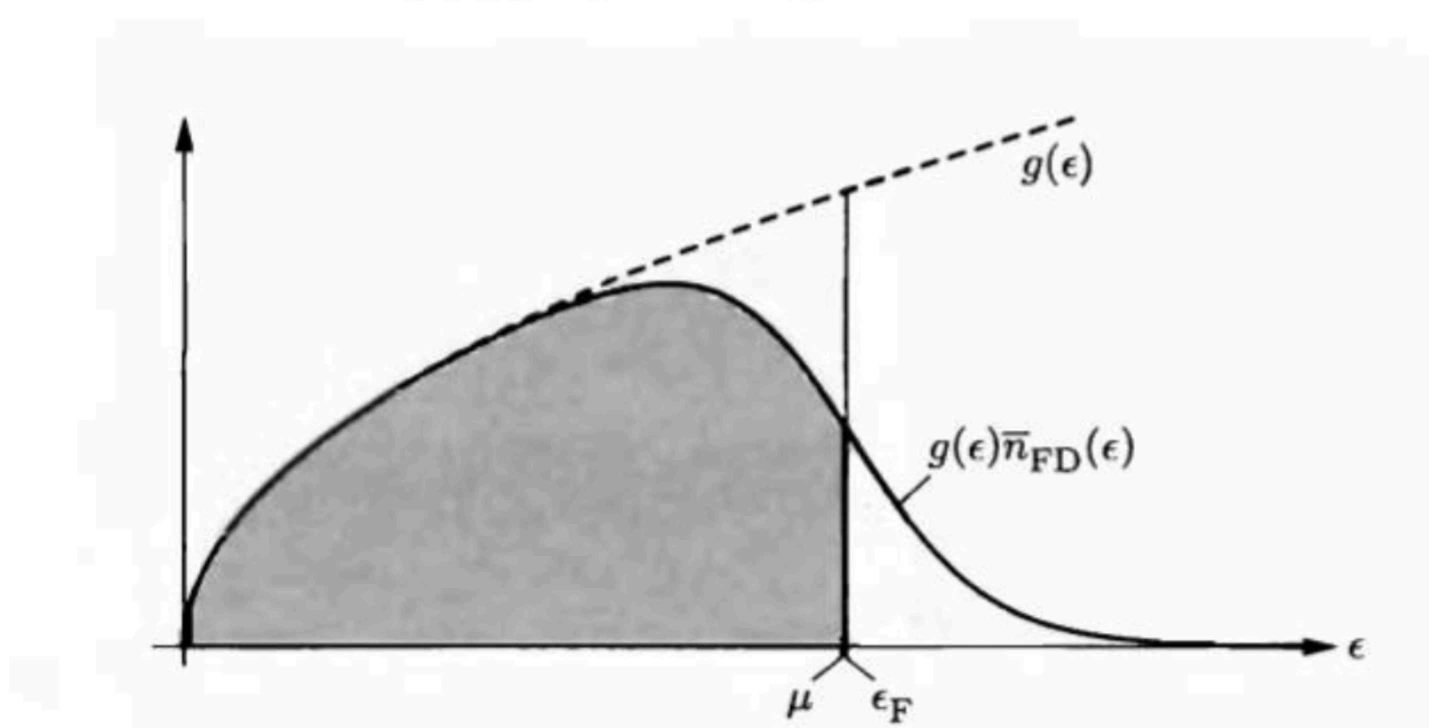
$$N = \int_0^{\infty} g(\epsilon)\overline{n}_{FD}(\epsilon) d\epsilon = \int_0^{\infty} g(\epsilon)\frac{1}{e^{(\epsilon-\mu)/kT}+1} d\epsilon$$

$\mu(T) \neq \epsilon_F$ except when $T = 0$.



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