Heat Engines

& Efficiency

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Heat Engines & Efficiency

What we Know:

- A heat engine is any device that absorbs heat and converts part of the energy into work

- The efficiency of a heat engine is defined by;

$$e \leqslant 1 - (T_c/T_h)$$

- e cannot be greater than 1.

- The smaller the T_c/T_h ratio is, the more efficient the engine can be.

The Carnot Engine

Derivation:

- Every engine has a working substance which absorbs heat and does work as well e.g. a gas.
- Want the gas to absorb some heat Q_h from a hot reservoir. Where here, the entropy reduces by Q_h/T_h in the reservoir and increases by Q_h/T_{gas} in the gas.
- Now, to avoid making any new entropy, we set $T_{gas} = T_h$ but we know heat doesn't flow between objects at the same temperature so we assume T_{gas} is slightly less than T_h.
- Want to keep it at this temperature by letting the gas expand as it absorbs heat (Isothermal)
- All that is left is to consider how to get the gas from one T to another and back with no Q added or taken out when the gas is at intermediate T (Adiabatic)

The Carnot Engine

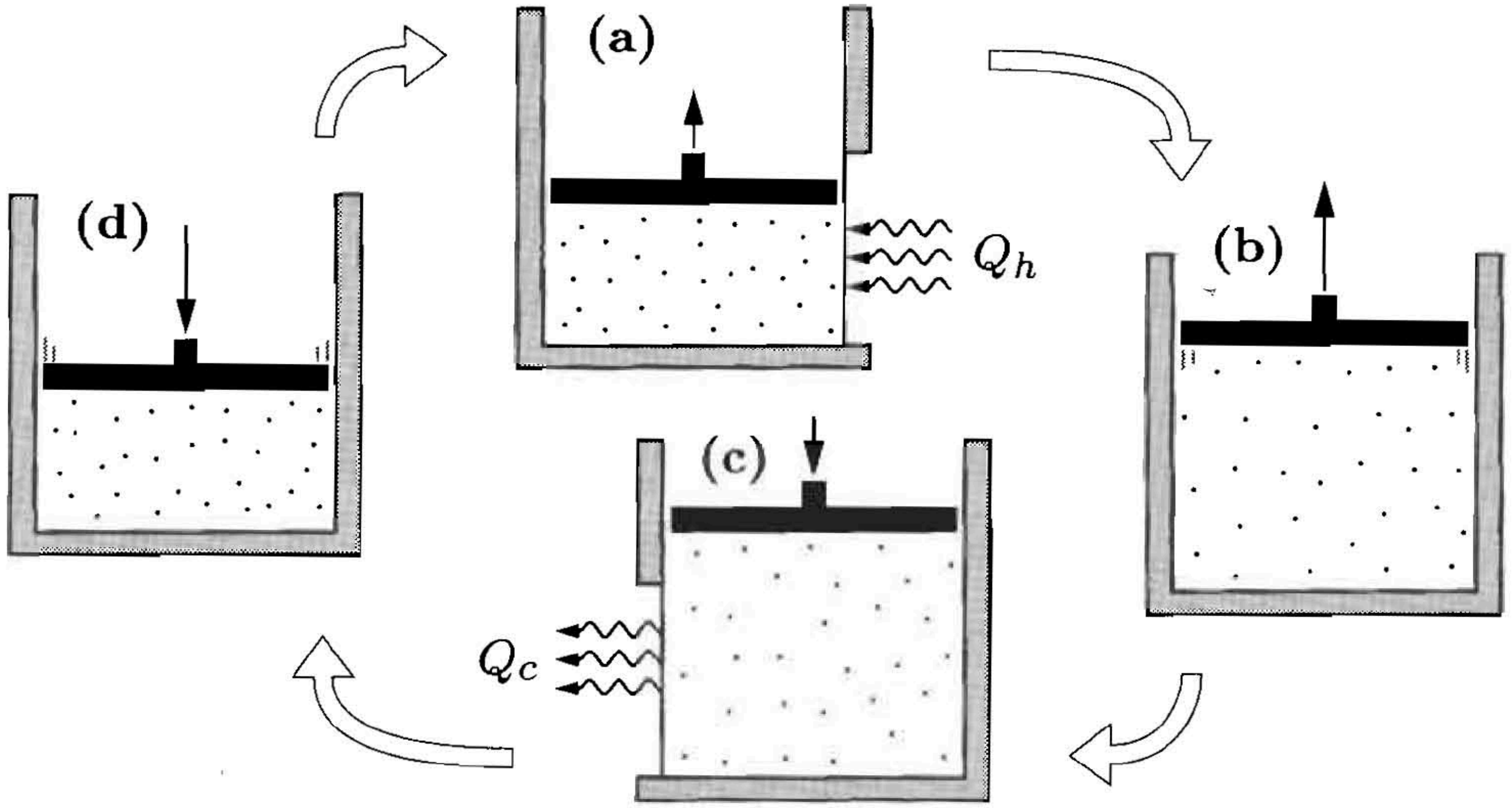
The Cycle:

A. Isothermal Expansion at $T < T_h$

B. Adiabatic expansion from T_h to T_c

C. Isothermal compression at T_c

D. Adiabatic compression from T_c to T_h



The Carnot Engine

Problem 4.5

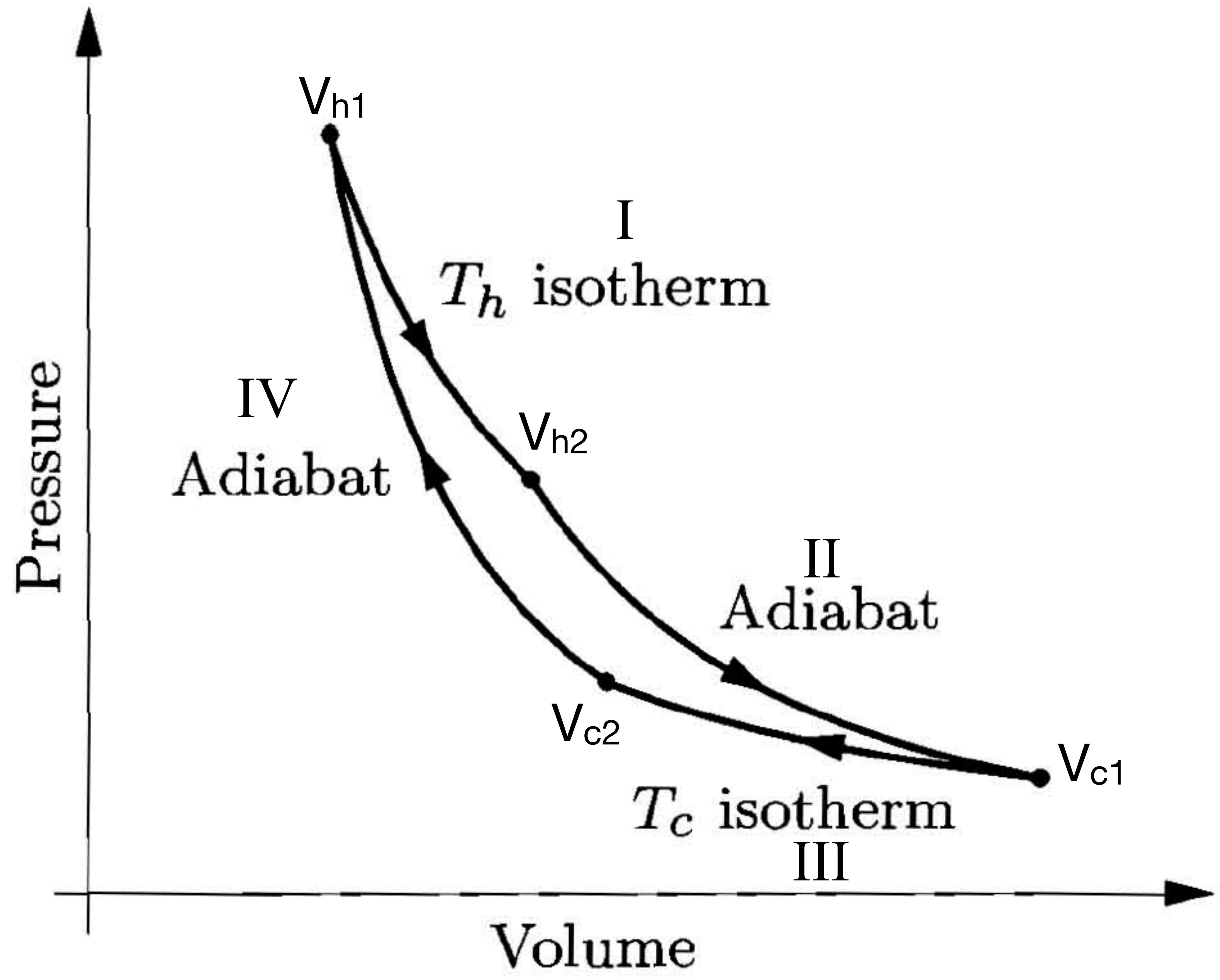
Prove directly that a Carnot engine, using an ideal gas as the working substance has an efficiency of $1 - (T_c/T_h)$.

- For the isothermal processes;

$$\Delta U = 0 \implies Q = W$$

- For the adiabatic processes;

$$Q = 0$$



I

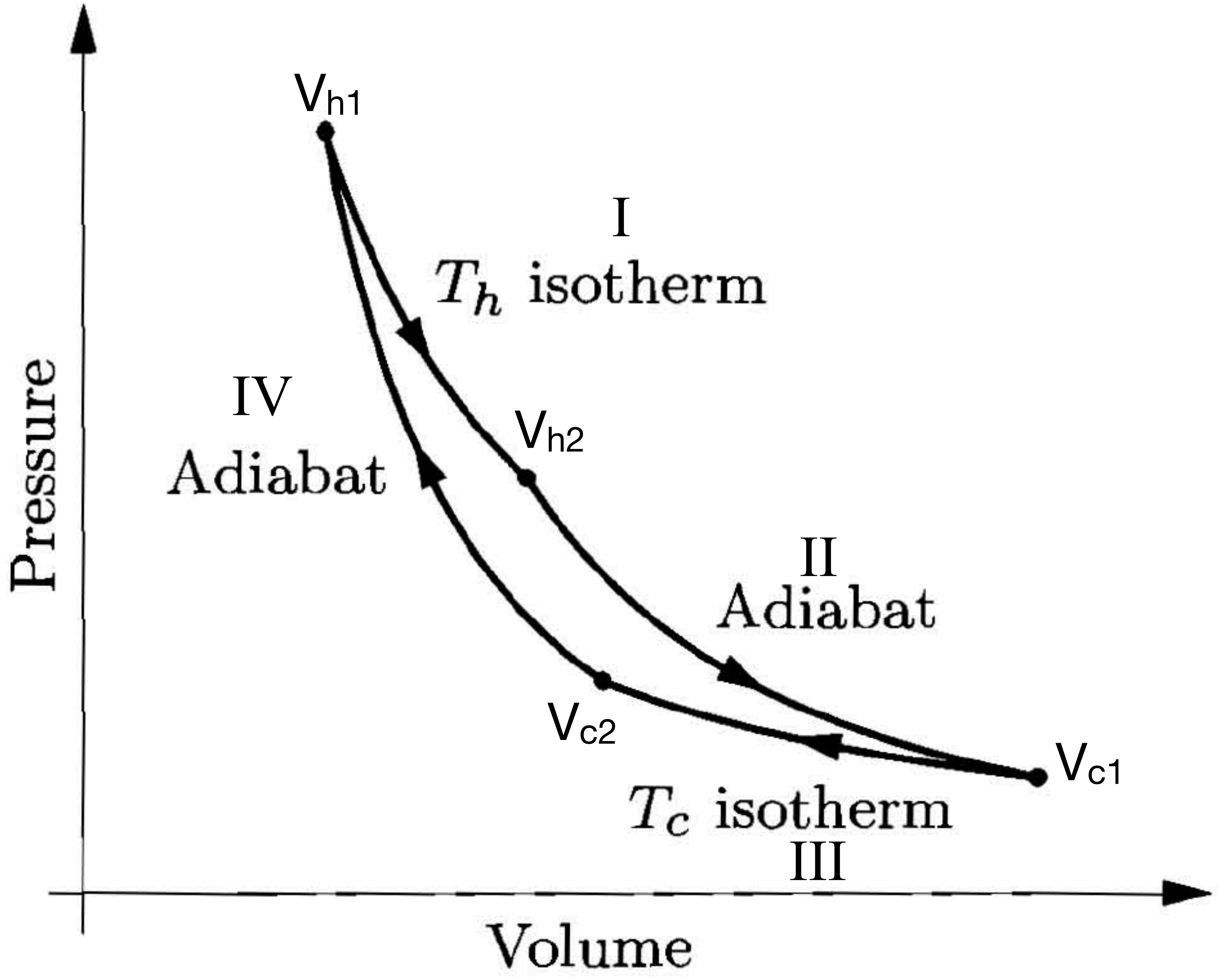
Isothermal Expansion

- $W = -P\Delta V;$
- $PV = nRT;$
- $P = nRT/V;$

$$\implies \quad W = nRT \int_{V_i}^{V_f} (1/V)\, dV$$

Hence,

- $W_h = nRT_h \ln(V_{h2}/V_{h1})$



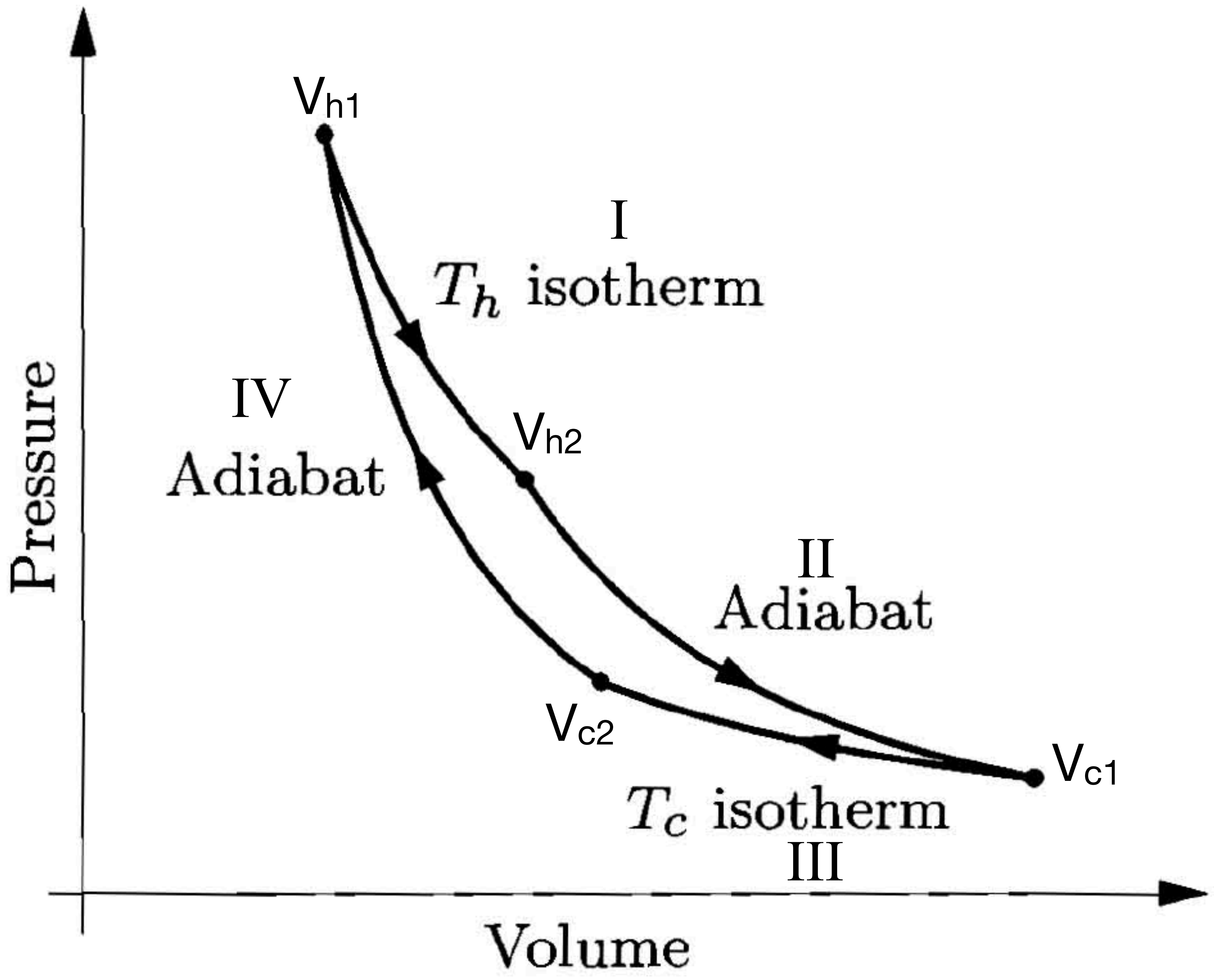
III

Isothermal Compression

- $W_c = nRT_c \ln(V_{c1}/V_{c2})$

Bear in mind that Q_c is defined as a positive number so this integral should be from a lower volume to higher volume for Q to be positive

$\implies$ *the integral is from V_{c2} to V_{c1}*



III

Bringing 1 and 3 together;

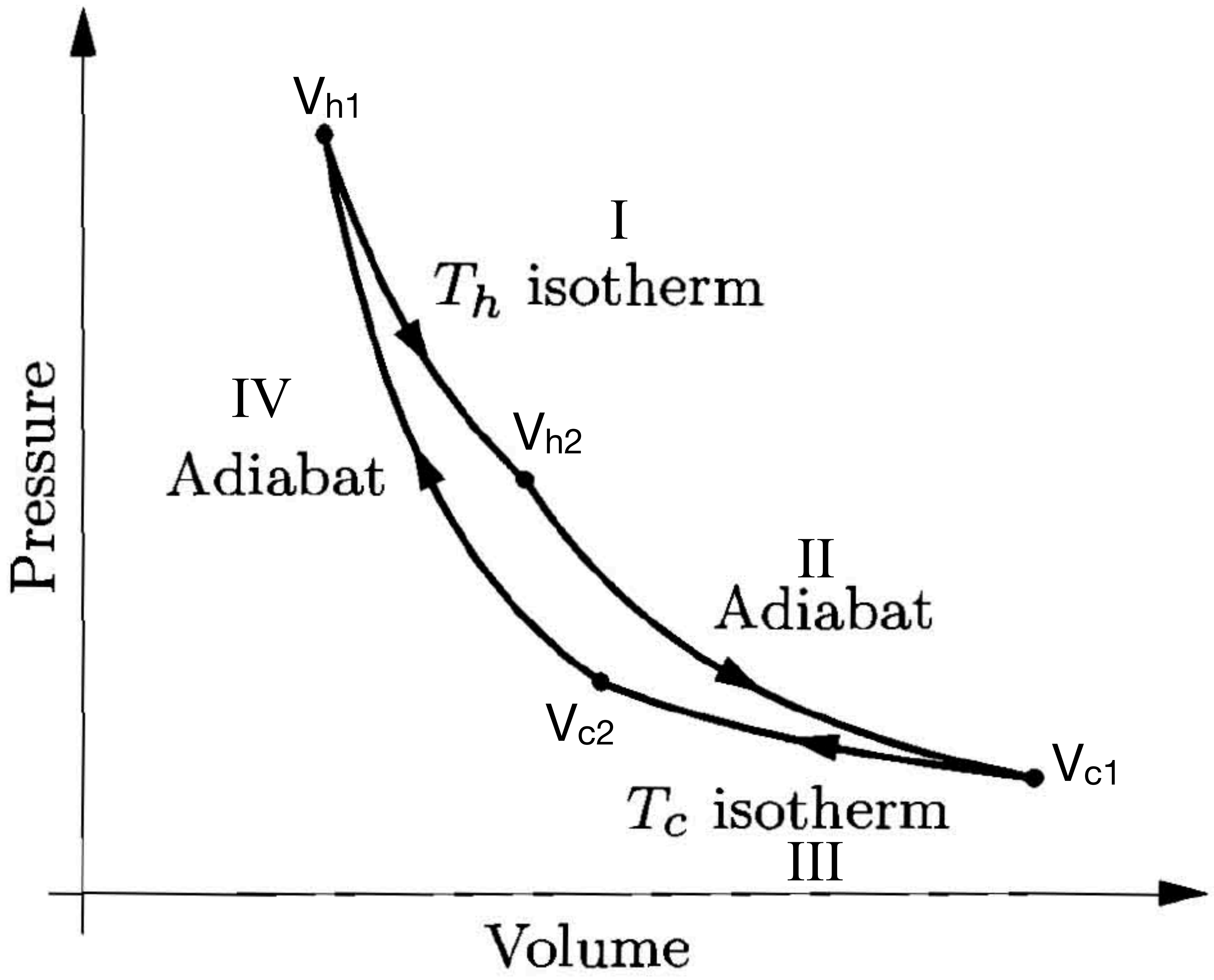
- $W_c = nRT_c \ln(V_{c1}/V_{c2}) = Q_c$
- $W_h = nRT_h \ln(V_{h2}/V_{h1}) = Qh$

We know;

- $e = 1 - (Q_c/Q_h)$

So,

$e = 1 - [nRT_c \ln(V_{c1}/V_{c2})/nRT_h \ln(V_{h2}/V_{h1})]$



II

Adiabatic Expansion

Using,

- $TV^{\gamma-1} = Const\,.$

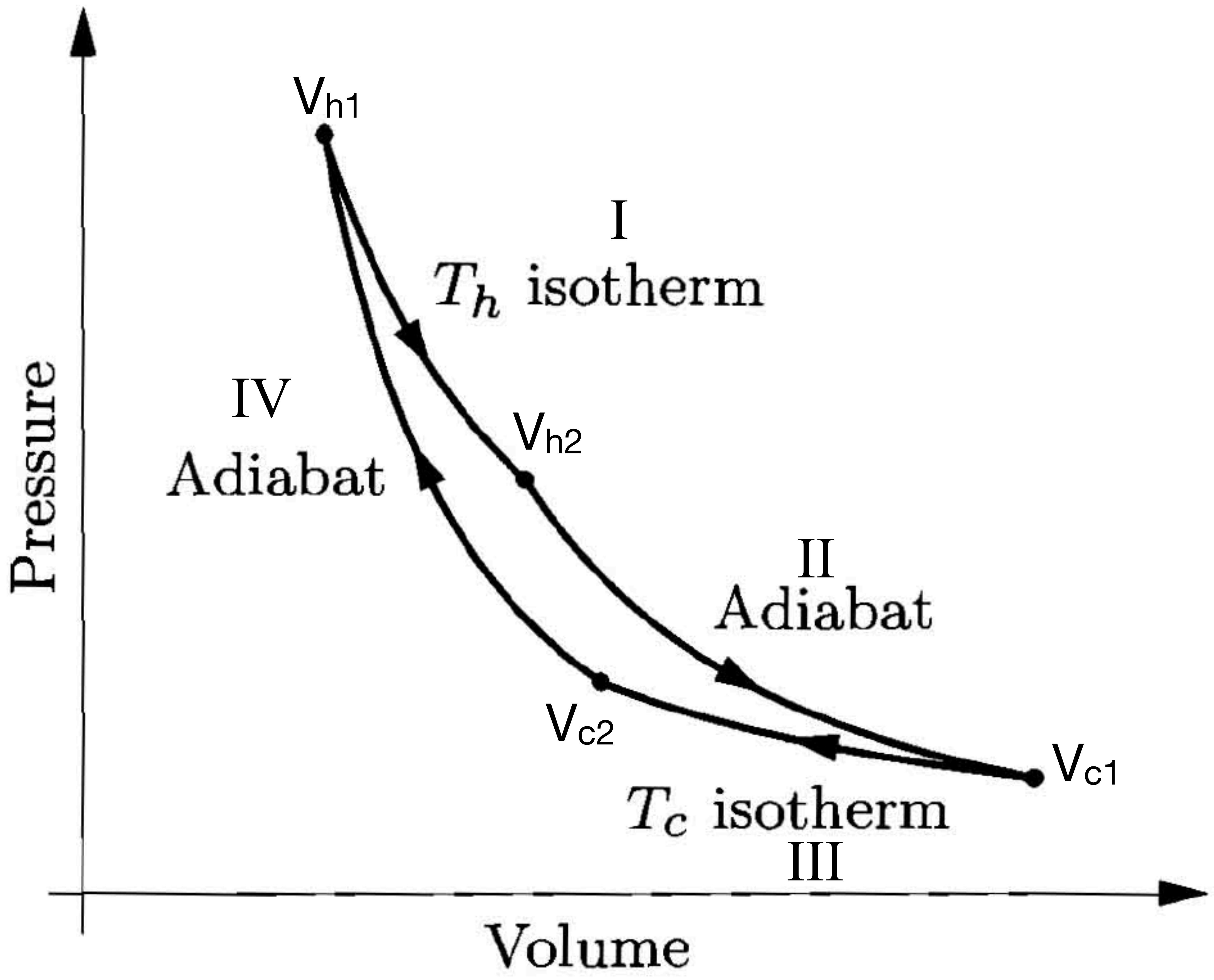
We can solve for,

- V_{c1}/V_{c2}

and substitute.

So,

- $T_h V_{h2}^{\gamma-1} = T_c V_{c1}^{\gamma-1}$



Adiabatic Expansion

Hence,

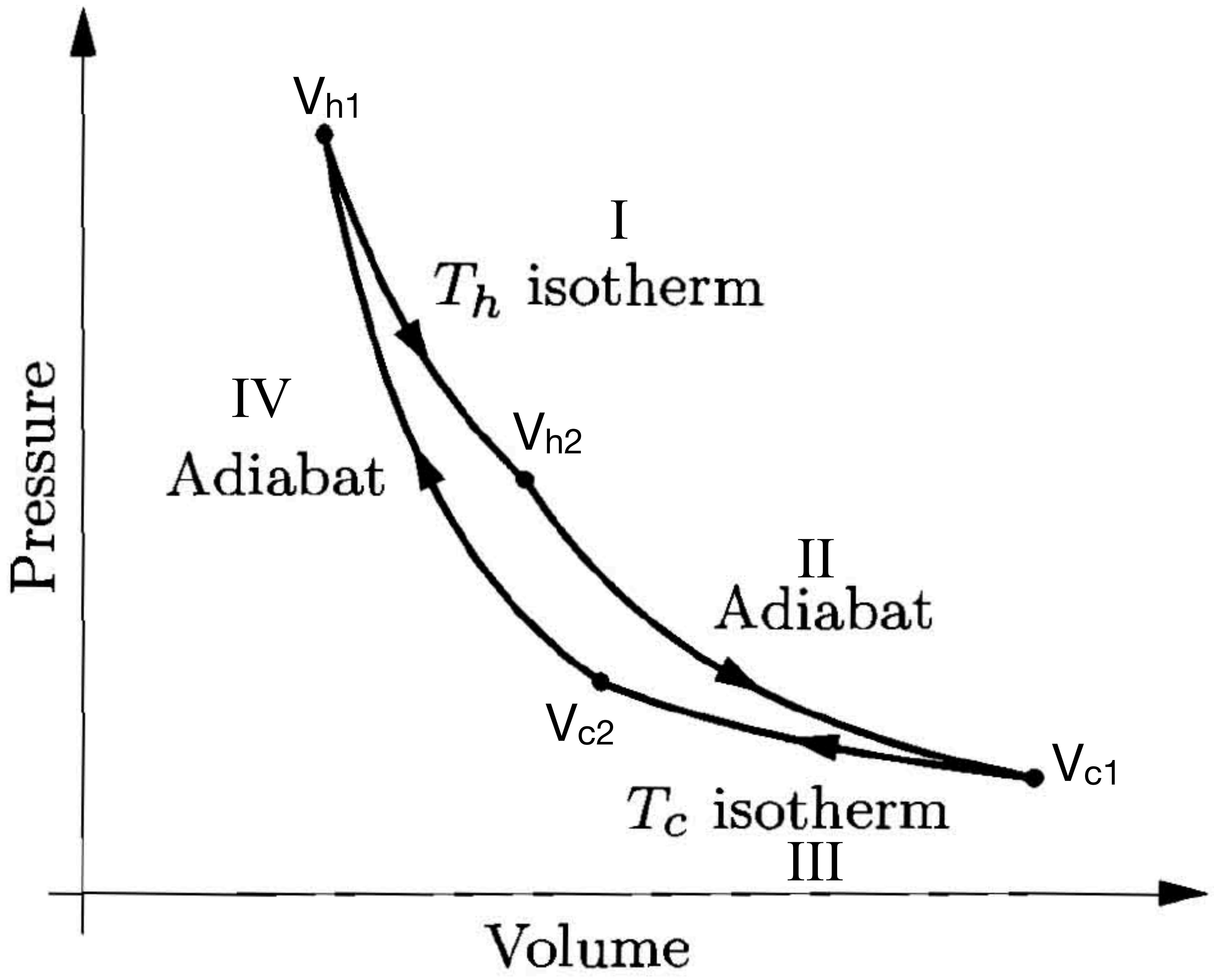
- $T_c/T_H = V_{h2}^{\gamma-1}/V_{c1}^{\gamma-1}$

IV

Adiabatic Compression

Here,

- $T_c V_{c2}^{\gamma-1} = T_h V_{h1}^{\gamma-1}$



Adiabatic Compression contd.

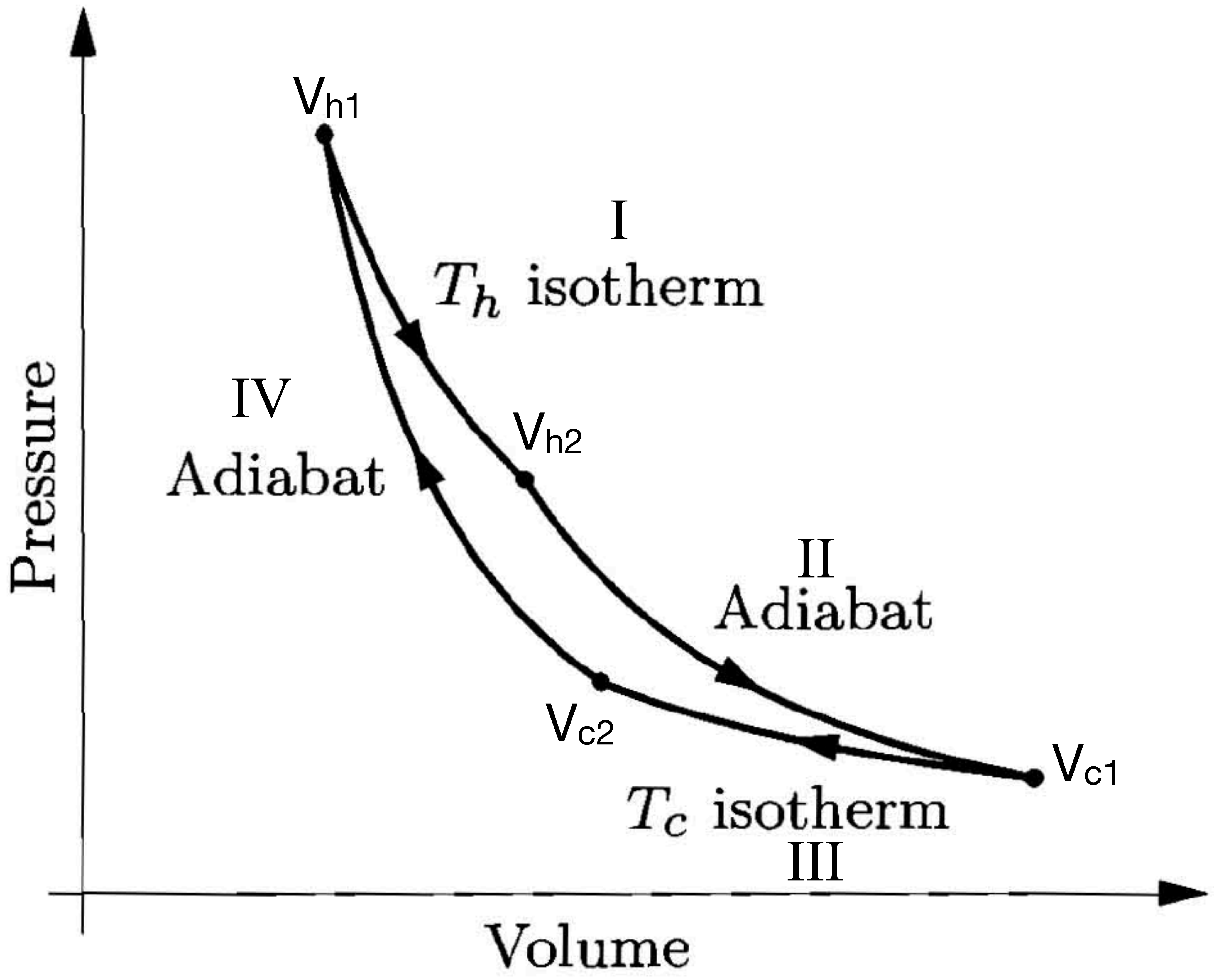
Which gives,

- $T_c/T_H = V_{h1}^{\gamma-1}/V_{c2}^{\gamma-1}$

Now, we can set III and IV equal to each other and solve for V_{c1}/V_{c2}.

- $T_c/T_H = V_{h1}^{\gamma-1}/V_{c2}^{\gamma-1} = V_{h2}^{\gamma-1}/V_{c1}^{\gamma-1}$

After some algebra, we find that =>



IV

Adiabatic Compression contd.

- $V_{c1}/V_{c2} = V_{h2}/V_{h1}$

Substituting

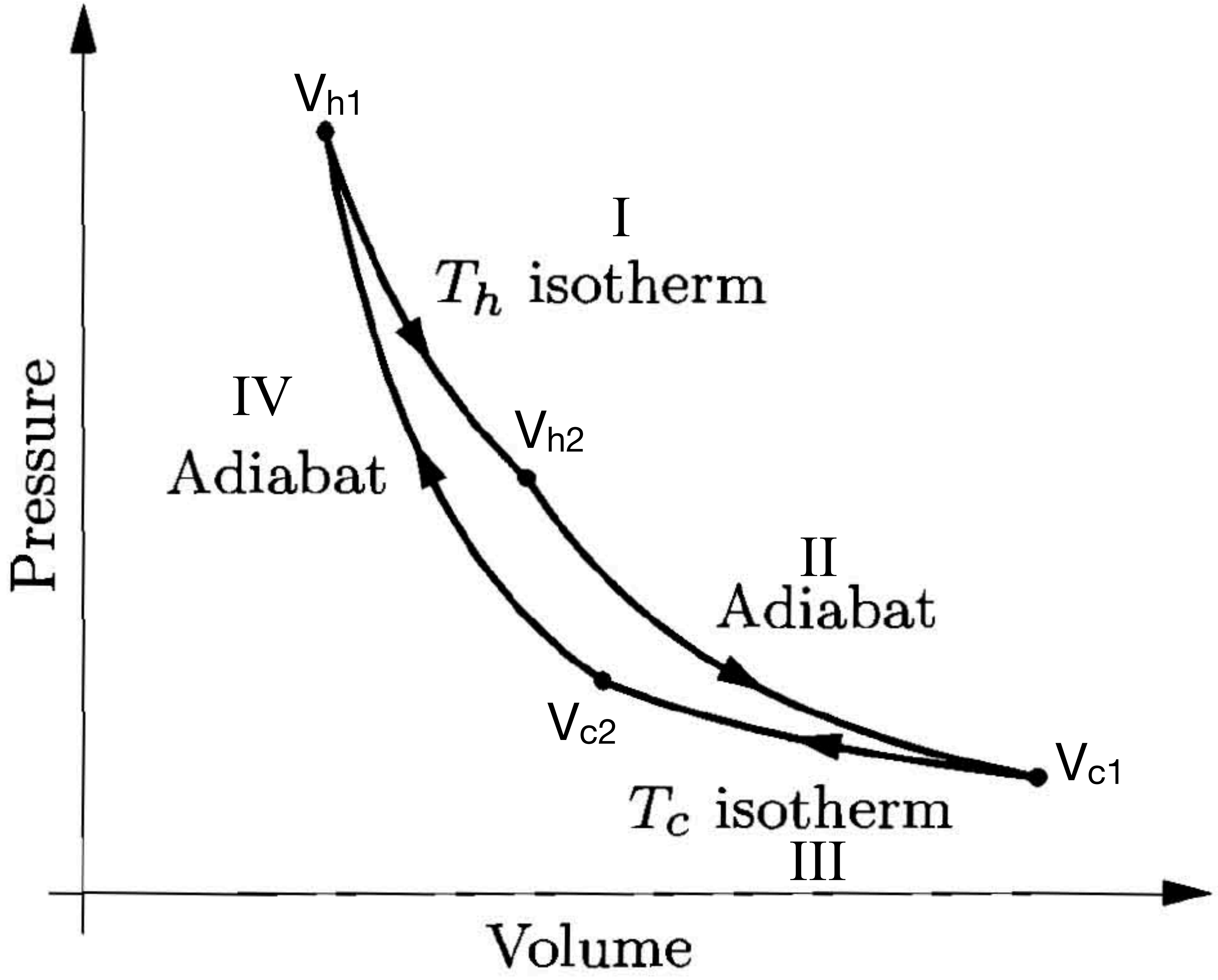
- V_{h2}/V_{h1} *for* V_{c1}/V_{c2},

In our equation for e gives

- $e = 1 - [nRT_c \ln(V_{h2}/V_{h1})/nRT_h \ln(V_{h2}/V_{h1})]$

And after some cancellation we find that;

- $e = 1 - (T_c/T_h)$



The Carnot Engine

However, Schroeder points out that as long as we know no new entropy has been created then the strict equality

$$Q_C/T_C \geqslant Q_h/T_h$$

And this result holds for non-ideal gases and other working substances.